Proof. $(\mathfrak{F};\mathfrak{P})$ is with join-closed core by corollary 4.96. $\mathfrak{F}$ is a meet-semilattice by corollary 4.107. So we can apply theorem 4.68. Then apply proposition 4.100. □

Proposition 4.154. If $\mathfrak{Z}$ is a complete lattice, then for every $S \in \mathscr{P}\mathfrak{F}$

$$\operatorname{Cor} \bigsqcap^{\mathfrak{F}} S = \bigsqcap^{\mathfrak{P}} \langle \operatorname{Cor} \rangle S.$$

Proof. By theorem 4.69. □

Corollary 4.155. If $\mathfrak{Z}$ is a complete lattice, then for every $S \in \mathscr{P}\mathfrak{P}$

$$\operatorname{Cor} \bigsqcap^{\mathfrak{F}} S = \bigsqcap^{\mathfrak{P}} S.$$

Proposition 4.156. Let $\mathfrak{Z}$ be a complete atomistic lattice. Then for every $a, b \in \mathfrak{F}$

$$\operatorname{Cor}(a \sqcup^{\mathfrak{F}} b) = \operatorname{Cor} a \sqcup^{\mathfrak{P}} \operatorname{Cor} b.$$

Proof. $(\mathfrak{F}; \mathfrak{P})$ is semifiltered by corollary 4.95. It is with finitely meet-close core by 4.97. $\mathfrak{F}$ is starrish by corollary 4.114. $\mathfrak{F}$ is complete by corollary 4.107. So we can apply theorem 4.71. Then apply proposition 4.100. □

Theorem 4.157. Let $\mathfrak{Z}$ be a complete boolean lattice. Then $(a \sqcap^{\mathfrak{F}} b)^* = a^* \sqcup^{\mathfrak{P}} b^*$ for every $a, b \in \mathfrak{F}$.

Proof. $(\mathfrak{F};\mathfrak{P})$ is a filtered (corollary 4.95) up-aligned complete lattice filtrator with finitely join-closed (theorem 4.25) co-separable core (theorem 4.73) which is a complete boolean lattice. Thus by the theorem 4.60

$$(a \sqcap^{\mathfrak{F}} b)^* = (a \sqcap^{\mathfrak{F}} b)^+ = \overline{\operatorname{Cor}(a \sqcap^{\mathfrak{F}} b)} = \overline{\operatorname{Cor} a \sqcap^{\mathfrak{P}} \operatorname{Cor} b} = \overline{\operatorname{Cor} a} \sqcup^{\mathfrak{P}} \overline{\operatorname{Cor} b} = a^+ \sqcup^{\mathfrak{P}} b^+ = a^* \sqcup^{\mathfrak{P}} b^*$$

(used propositions 4.149, 4.153). □

Theorem 4.158. Let $\mathfrak{Z}$ be a complete atomistic boolean lattice. Then $(a \sqcup^{\mathfrak{F}} b)^* = a^* \sqcap^{\mathfrak{P}} b^*$ for every $a, b \in \mathfrak{F}$.

Proof. $(\mathfrak{F};\mathfrak{P})$ is a filtered (corollary 4.95), distributive (corollary 4.114) complete lattice filtrator (corollary 4.107), with finitely meet-closed core (proposition 4.97), with separable core (theorem 4.112). So we can apply the theorem 4.72. □

4.3.21 Complementive Filters and Factoring by a Filter

Definition 4.159. Let $\mathfrak{A}$ be a meet-semilattice and $\mathcal{A} \in \mathfrak{A}$. The relation $\sim$ on $\mathfrak{A}$ is defined by the formula

$$\forall X, Y \in \mathfrak{A}: (X \sim Y \Leftrightarrow X \sqcap^{\mathfrak{A}} \mathcal{A} = Y \sqcap^{\mathfrak{A}} \mathcal{A}).$$

Proposition 4.160. The relation $\sim$ is an equivalence relation.

Proof.

Reflexivity. Obvious.

Symmetry. Obvious.

Transitivity. Obvious. □

Definition 4.161. When $X, Y \in \mathfrak{Z}$ and $\mathcal{A} \in \mathfrak{F}$ we define $X \sim Y \Leftrightarrow \uparrow X \sim \uparrow Y$.

Theorem 4.162. Let $\mathfrak{Z}$ be a distributive lattice [TODO: Generalize for meet-semilattices?], $\mathcal{A} \in \mathfrak{F}$. Then for every $X, Y \in \mathfrak{Z}$

$$X \sim Y \Leftrightarrow \exists A \in \mathcal{A}: X \sqcap^{\mathfrak{Z}} A = Y \sqcap^{\mathfrak{Z}} A.$$