Knowing core part and edge part or dual core part and dual edge part of a filter, the filter can be restored by the formulas:

$$a = \operatorname{Cor} a \sqcup^{\mathfrak{A}} \operatorname{Edg} a \quad \text{and} \quad a = \operatorname{Cor}' a \sqcup^{\mathfrak{A}} \operatorname{Edg}' a.$$

4.2.10 Core Part and Atomic Elements

Proposition 4.67. Let $(\mathfrak{A}; \mathfrak{Z})$ be a filtrator with join-closed core and $\mathfrak{Z}$ be an atomistic lattice. Then for every $a \in \mathfrak{A}$ such that $\operatorname{Cor}' a$ exists we have

$$\operatorname{Cor}' a = \bigsqcup^{\mathfrak{Z}} \{x \mid x \text{ is an atom of } \mathfrak{Z}, x \sqsubseteq a\}.$$

Proof.

$$\begin{aligned} \operatorname{Cor}' a &= \\ \bigsqcup^{\mathfrak{Z}} \{A \in \mathfrak{Z} \mid A \sqsubseteq a\} &= \\ \bigsqcup^{\mathfrak{Z}} \left\{ \bigsqcup^{\mathfrak{Z}} \operatorname{atoms}^{\mathfrak{Z}} A \mid A \in \mathfrak{Z}, A \sqsubseteq a \right\} &= \\ \bigsqcup^{\mathfrak{Z}} \bigcup \{\operatorname{atoms}^{\mathfrak{Z}} A \mid A \in \mathfrak{Z}, A \sqsubseteq a\} &= \\ \bigsqcup^{\mathfrak{Z}} \{x \mid x \text{ is an atom of } \mathfrak{Z}, x \sqsubseteq a\}. \end{aligned}$$

□

4.2.11 Distributivity of Core Part over Lattice Operations

Theorem 4.68. If $(\mathfrak{A}; \mathfrak{Z})$ is a join-closed filtrator and $\mathfrak{A}$ is a meet-semilattice and $\mathfrak{Z}$ is a complete lattice, then for every $a, b \in \mathfrak{A}$

$$\operatorname{Cor}'(a \sqcap^{\mathfrak{A}} b) = \operatorname{Cor}' a \sqcap^{\mathfrak{Z}} \operatorname{Cor}' b.$$

Proof. From theorem conditions it follows that $\operatorname{Cor}'(a \sqcap^{\mathfrak{A}} b)$ exists.

We have $\operatorname{Cor}' p \sqsubseteq p$ for every $p \in \mathfrak{A}$ because our filtrator is with join-closed core.

Obviously $\operatorname{Cor}'(a \sqcap^{\mathfrak{A}} b) \sqsubseteq \operatorname{Cor}' a$ and $\operatorname{Cor}'(a \sqcap^{\mathfrak{A}} b) \sqsubseteq \operatorname{Cor}' b$.

If $x \sqsubseteq \operatorname{Cor}' a$ and $x \sqsubseteq \operatorname{Cor}' b$ for some $x \in \mathfrak{Z}$ then $x \sqsubseteq a$ and $x \sqsubseteq b$, thus $x \sqsubseteq a \sqcap^{\mathfrak{A}} b$ and $x \sqsubseteq \operatorname{Cor}'(a \sqcap^{\mathfrak{A}} b)$. □

Theorem 4.69. If $(\mathfrak{A}; \mathfrak{Z})$ is a join-closed filtrator and both $\mathfrak{A}$ and $\mathfrak{Z}$ are complete lattices, then for every $S \in \mathscr{P}\mathfrak{A}$

$$\operatorname{Cor}' \bigsqcap^{\mathfrak{A}} S = \bigsqcap^{\mathfrak{Z}} \langle \operatorname{Cor}' \rangle S.$$

Proof. From theorem conditions it follows that $\operatorname{Cor}' \bigsqcap^{\mathfrak{A}} S$ exists.

We have $\operatorname{Cor}' p \sqsubseteq p$ for every $p \in \mathfrak{A}$ because our filtrator is with join-closed core.

Obviously $\operatorname{Cor}' \bigsqcap^{\mathfrak{A}} S \sqsubseteq \operatorname{Cor}' a$ for every $a \in S$.

If $x \sqsubseteq \operatorname{Cor}' a$ for every $a \in S$ for some $x \in \mathfrak{Z}$ then $x \sqsubseteq a$, thus $x \sqsubseteq \bigsqcap^{\mathfrak{A}} S$ and $x \sqsubseteq \operatorname{Cor}' \bigsqcap^{\mathfrak{A}} S$. □

Corollary 4.70. If $(\mathfrak{A}; \mathfrak{Z})$ is a join-closed filtrator and both $\mathfrak{A}$ and $\mathfrak{Z}$ are complete lattices, then for every $S \in \mathscr{P}\mathfrak{Z}$

$$\operatorname{Cor}' \bigsqcap^{\mathfrak{A}} S = \bigsqcap^{\mathfrak{Z}} S.$$

Theorem 4.71. Let $(\mathfrak{A}; \mathfrak{Z})$ be a semifiltered down-aligned filtrator with finitely meet-closed core $\mathfrak{Z}$ which is a complete atomistic lattice and $\mathfrak{A}$ is a complete starrish lattice, then $\operatorname{Cor}'(a \sqcup^{\mathfrak{A}} b) = \operatorname{Cor}' a \sqcup^{\mathfrak{Z}} \operatorname{Cor}' b$ for every $a, b \in \mathfrak{A}$.