(used the lemma and theorem 4.43). □

Corollary 4.61. If $(\mathfrak{A};\mathfrak{Z})$ is a filtered up-aligned complete lattice filtrator with co-separable core which is a complete boolean lattice, then $a^+ \in \mathfrak{Z}$ for every $a \in \mathfrak{A}$.

Theorem 4.62. If $(\mathfrak{A};\mathfrak{Z})$ is a filtered complete lattice filtrator with down-aligned, finitely meet-closed, separable core which is a complete boolean lattice, then $a^* = \overline{\operatorname{Cor} a} = \overline{\operatorname{Cor}' a}$ for every $a \in \mathfrak{A}$.

Proof. Our filtrator is with join-closed core (theorem 4.25). $a^* = \bigsqcup^{\mathfrak{A}} \{c \in \mathfrak{A} \mid c \sqcap^{\mathfrak{A}} a = 0^{\mathfrak{A}}\}$. But $c \sqcap^{\mathfrak{A}} a = 0^{\mathfrak{A}} \Rightarrow \exists C \in \operatorname{up} c : C \sqcap^{\mathfrak{A}} a = 0^{\mathfrak{A}}$. So

$$\begin{aligned}
a^* &= \\
\bigsqcup^{\mathfrak{A}} \{C \in \mathfrak{Z} \mid C \sqcap^{\mathfrak{A}} a = 0^{\mathfrak{A}}\} &= \\
\bigsqcup^{\mathfrak{A}} \{C \in \mathfrak{Z} \mid a \sqsubseteq \overline{C}\} &= \\
\bigsqcup^{\mathfrak{A}} \{\overline{C} \mid C \in \mathfrak{Z}, a \sqsubseteq C\} &= \\
\bigsqcup^{\mathfrak{A}} \{\overline{C} \mid C \in \operatorname{up} a\} &= \\
\bigsqcup^{\mathfrak{Z}} \{\overline{C} \mid C \in \operatorname{up} a\} &= \\
\overline{\bigsqcap^{\mathfrak{Z}} \{C \mid C \in \operatorname{up} a\}} &= \\
\overline{\bigsqcap^{\mathfrak{Z}} \operatorname{up} a} &= \\
\overline{\operatorname{Cor} a}
\end{aligned}$$

(used theorem 4.43).

$\operatorname{Cor} a = \operatorname{Cor}' a$ by theorem 4.34. □

Corollary 4.63. If $(\mathfrak{A};\mathfrak{Z})$ is a filtered down-aligned and up-aligned complete lattice filtrator with finitely meet-closed, separable and co-separable core which is a complete boolean lattice, then $a^* = a^+$ for every $a \in \mathfrak{A}$.

Proof. Comparing two last theorems. □

Theorem 4.64. If $(\mathfrak{A};\mathfrak{Z})$ is a complete lattice filtrator with join-closed separable core which is a complete lattice, then $a^* \in \mathfrak{Z}$ for every $a \in \mathfrak{A}$.

Proof. $\{c \in \mathfrak{A} \mid c \sqcap^{\mathfrak{A}} a = 0^{\mathfrak{A}}\} \supseteq \{A \in \mathfrak{Z} \mid A \sqcap^{\mathfrak{A}} a = 0^{\mathfrak{A}}\}$; consequently $a^* \sqsupseteq \bigsqcup^{\mathfrak{A}} \{A \in \mathfrak{Z} \mid A \sqcap^{\mathfrak{A}} a = 0^{\mathfrak{A}}\}$.

But if $c \in \{c \in \mathfrak{A} \mid c \sqcap^{\mathfrak{A}} a = 0^{\mathfrak{A}}\}$ then there exists $A \in \mathfrak{Z}$ such that $A \sqsupseteq c$ and $A \sqcap^{\mathfrak{A}} a = 0^{\mathfrak{A}}$ that is $A \in \{A \in \mathfrak{Z} \mid A \sqcap^{\mathfrak{A}} a = 0^{\mathfrak{A}}\}$. Consequently $a^* \sqsubseteq \bigsqcup^{\mathfrak{A}} \{A \in \mathfrak{Z} \mid A \sqcap^{\mathfrak{A}} a = 0^{\mathfrak{A}}\}$.

We have $a^* = \bigsqcup^{\mathfrak{A}} \{A \in \mathfrak{Z} \mid A \sqcap^{\mathfrak{A}} a = 0^{\mathfrak{A}}\} = \bigsqcup^{\mathfrak{Z}} \{A \in \mathfrak{Z} \mid A \sqcap^{\mathfrak{A}} a = 0^{\mathfrak{A}}\} \in \mathfrak{Z}$. □

Theorem 4.65. If $(\mathfrak{A};\mathfrak{Z})$ is an up-aligned filtered complete lattice filtrator with co-separable core which is a complete boolean lattice, then a^+ is dual pseudocomplement of a, that is

$$a^+ = \min \{c \in \mathfrak{A} \mid c \sqcup^{\mathfrak{A}} a = 1^{\mathfrak{A}}\}$$

for every $a \in \mathfrak{A}$.

Proof. Our filtrator is with join-closed core (theorem 4.25). It's enough to prove that $a^+ \sqcup^{\mathfrak{A}} a = 1^{\mathfrak{A}}$. But $a^+ \sqcup^{\mathfrak{A}} a = \overline{\operatorname{Cor} a} \sqcup^{\mathfrak{A}} a \sqsupseteq \overline{\operatorname{Cor} a} \sqcup^{\mathfrak{A}} \operatorname{Cor} a = \overline{\operatorname{Cor} a} \sqcup^{\mathfrak{Z}} \operatorname{Cor} a = 1^{\mathfrak{A}}$ (used the theorem 4.29 and the fact that our filtrator is filtered). □

Definition 4.66. The *edge part* of an element $a \in \mathfrak{A}$ is $\operatorname{Edg} a = a \setminus \operatorname{Cor} a$, the *dual edge part* is $\operatorname{Edg}' a = a \setminus \operatorname{Cor}' a$.