Thus $\operatorname{uncurry}(\operatorname{uncurry} \circ S)(i;\ (x;\ y)) = (\operatorname{uncurry}(\operatorname{uncurry} S))((i;\ x);\ y)$ and thus evidently $\operatorname{uncurry}(\operatorname{uncurry} \circ S) \sim \operatorname{uncurry}(\operatorname{uncurry} S)$. □

Theorem 3.106. concat is an infinitely associative function.

Proof. $\operatorname{concat}(\llbracket x \rrbracket) = x$ for a function x taking an ordinal number of argument is obvious. It is remained to prove

$$\operatorname{concat}(\operatorname{concat} \circ S) = \operatorname{concat}(\operatorname{concat} S);$$

We have, using the lemmas, $\operatorname{concat}(\operatorname{concat} \circ S) \sim \operatorname{uncurry}(\operatorname{concat} \circ S) \sim$ (by lemma 3.104)$\sim \operatorname{uncurry}(\operatorname{uncurry} \circ S) \sim \operatorname{uncurry}(\operatorname{uncurry} S) \sim \operatorname{uncurry}(\operatorname{concat} S) \sim \operatorname{concat}(\operatorname{concat} S)$. □

Corollary 3.107. Ordinated product is an infinitely associative function.