Proof. Let f be bijective. Then $f\circ f^{\dagger}\sqsubseteq 1_{\operatorname{Dst} f}$, $f^{\dagger}\circ f\sqsupseteq 1_{\operatorname{Src} f}$, $f^{\dagger}\circ f\sqsubseteq 1_{\operatorname{Src} f}$, $f\circ f^{\dagger}\sqsupseteq 1_{\operatorname{Dst} f}$. Thus $f\circ f^{\dagger}=1_{\operatorname{Dst} f}$ and $f^{\dagger}\circ f=1_{\operatorname{Src} f}$ that is $f^{\dagger}$ is an inverse of f. □

[TODO: Below require that Mor-sets are complete lattices.]

Definition 3.61. A morphism f of a partially ordered category is *metamonovalued* when $(\bigsqcap G)\circ f=\bigsqcap_{g\in G}(g\circ f)$ whenever G is a set of morphisms with a suitable domain and image.

Definition 3.62. A morphism f of a partially ordered category is *metainjective* when $f\circ(\bigsqcap G)=\bigsqcap_{g\in G}(f\circ g)$ whenever G is a set of morphisms with a suitable domain and image.

Obvious 3.63. Metamonovaluedness and metainjectivity are dual to each other.

Definition 3.64. A morphism f of a partially ordered category is *metacomplete* when $f\circ(\bigsqcup G)=\bigsqcap_{g\in G}(f\circ g)$ whenever G is a set of morphisms with a suitable domain and image.

Definition 3.65. A morphism f of a partially ordered category is *co-metacomplete* when $(\bigsqcup G)\circ f=\bigsqcap_{g\in G}(g\circ f)$ whenever G is a set of morphisms with a suitable domain and image.

3.6 Partitioning

Definition 3.66. Let $\mathfrak{A}$ be a complete lattice. *Torning* of an element $a\in\mathfrak{A}$ is a set $S\in\mathscr{P}\mathfrak{A}\setminus\{0\}$ such that

$$\bigsqcup S=a\quad\text{and}\quad\forall x,y\in S:(x\neq y\Rightarrow x\asymp y).$$

Definition 3.67. Let $\mathfrak{A}$ be a complete lattice. *Weak partition* of an element $a\in\mathfrak{A}$ is a set $S\in\mathscr{P}\mathfrak{A}\setminus\{0\}$ such that

$$\bigsqcup S=a\quad\text{and}\quad\forall x\in S:x\asymp\bigsqcup(S\setminus\{x\}).$$

Definition 3.68. Let $\mathfrak{A}$ be a complete lattice. *Strong partition* of an element $a\in\mathfrak{A}$ is a set $S\in\mathscr{P}\mathfrak{A}\setminus\{0\}$ such that

$$\bigsqcup S=a\quad\text{and}\quad\forall A,B\in\mathscr{P}S:\left(A\asymp B\Rightarrow\bigsqcup A\asymp\bigsqcup B\right).$$

Obvious 3.69.

1. Every strong partition is a weak partition.
2. Every weak partition is a torning.

3.7 A proposition about binary relations

Proposition 3.70. Let f, g, h be binary relations. Then $g\circ f\not\asymp h\Leftrightarrow g\not\asymp h\circ f^{-1}$.

Proof.

$$\begin{aligned}
g\circ f\not\asymp h &\Leftrightarrow\\
\exists a,c: a\,((g\circ f)\cap h)\,c &\Leftrightarrow\\
\exists a,c: (a\,(g\circ f)\,c\wedge a\,h\,c) &\Leftrightarrow\\
\exists a,b,c: (a\,f\,b\wedge b\,g\,c\wedge a\,h\,c) &\Leftrightarrow\\
\exists b,c: (b\,g\,c\wedge b\,(h\circ f^{-1})\,c) &\Leftrightarrow\\
\exists b,c: (b\,(g\cap(h\circ f^{-1}))\,c) &\Leftrightarrow\\
g\not\asymp h\circ f^{-1}. &
\end{aligned}$$

□