2. $\forall a, b \in \mathfrak{A}: (f a \sqsubseteq f b \Rightarrow a \sqsubseteq b)$.
3. $\forall a, b \in \mathfrak{A}: (a \sqsubset b \Rightarrow f a \sqsubset f b)$.
4. $\forall a, b \in \mathfrak{A}: (a \sqsubset b \Rightarrow f a \neq f b)$.
5. $\forall a, b \in \mathfrak{A}: (a \sqsubset b \Rightarrow f a \not\sqsupseteq f b)$.
6. $\forall a, b \in \mathfrak{A}: (f a \sqsubseteq f b \Rightarrow a \not\sqsupset b)$.

Proof.

(1)⇒(3). Let $a, b \in \mathfrak{A}$. Let $f a = f b \Rightarrow a = b$. Let $a \sqsubset b$. $f a \neq f b$ because $a \neq b$. $f a \sqsubseteq f b$ because $a \sqsubseteq b$. So $f a \sqsubset f b$.

(2)⇒(1). Let $a, b \in \mathfrak{A}$. Let $f a \sqsubseteq f b \Rightarrow a \sqsubseteq b$. Let $f a = f b$. Then $a \sqsubseteq b$ and $b \sqsubseteq a$ and consequently $a = b$.

(3)⇒(2). Let $\forall a, b \in \mathfrak{A}: (a \sqsubset b \Rightarrow f a \sqsubset f b)$. Let $a \not\sqsubseteq b$. Then $a \sqsupset a \sqcap b$. So $f a \sqsupset f(a \sqcap b)$. If $f a \sqsubseteq f b$ then $f a \sqsubseteq f\,(a \sqcap b)$ what is a contradiction.

(3)⇒(5)⇒(4). Obvious.

(4)⇒(3). Because $a \sqsubset b \Rightarrow a \sqsubseteq b \Rightarrow f a \sqsubseteq f b$.

(5)⇔(6). Obvious. □

3.1.2 Separation subsets and full stars

Definition 3.7. $\partial_Y a = \{x \in Y \mid x \not\asymp a\}$ for an element a of a poset $\mathfrak{A}$ and $Y \in \mathscr{P}\mathfrak{A}$.

Definition 3.8. *Full star* of $a \in \mathfrak{A}$ is $\star a = \partial_{\mathfrak{A}}\, a$.

Proposition 3.9. If $\mathfrak{A}$ is a meet-semilattice, then $\star$ is a straight monotone map.

Proof. Monotonicity is obvious. Let $\star a \not\sqsubseteq \star\,(a \sqcap b)$. Then it exists $x \in \star a$ such that $x \notin \star(a \sqcap b)$. So $x \sqcap a \notin \star b$ but $x \sqcap a \in \star a$ and consequently $\star a \not\sqsubseteq \star\, b$. □

Definition 3.10. A *separation subset* of a poset $\mathfrak{A}$ is such its subset Y that

$$\forall a, b \in \mathfrak{A}: (\partial_Y a = \partial_Y b \Rightarrow a = b).$$

Definition 3.11. I call *separable* such poset that $\star$ is an injection.

Obvious 3.12. A poset is separable iff it has a separation subset.

Definition 3.13. A poset $\mathfrak{A}$ has *disjunction property of Wallman* iff for any $a, b \in \mathfrak{A}$ either $b \sqsubseteq a$ or there exists a non-least element $c \sqsubseteq b$ such that $a \asymp c$.

Theorem 3.14. For a meet-semilattice with least element the following statements are equivalent:

1. $\mathfrak{A}$ is separable.
2. $\forall a, b \in \mathfrak{A}: (\star a \sqsubseteq \star b \Rightarrow a \sqsubseteq b)$.
3. $\forall a, b \in \mathfrak{A}: (a \sqsubset b \Rightarrow \star a \sqsubset \star b)$.
4. $\forall a, b \in \mathfrak{A}: (a \sqsubset b \Rightarrow \star a \neq \star b)$.
5. $\forall a, b \in \mathfrak{A}: (a \sqsubset b \Rightarrow \star a \not\sqsupseteq \star\, b)$.
6. $\forall a, b \in \mathfrak{A}: (\star a \sqsubseteq \star b \Rightarrow a \not\sqsupset b)$.
7. $\mathfrak{A}$ conforms to Wallman's disjunction property.
8. $\forall a, b \in \mathfrak{A}: (a \sqsubset b \Rightarrow \exists c \in \mathfrak{A} \setminus \{0\}: (c \asymp a \wedge c \sqsubseteq b))$.