Proof. $1\setminus^{*}b=\min\{z\in\mathfrak{A}\mid 1\sqsubseteq b\sqcup z\}=\min\{z\in\mathfrak{A}\mid 1=b\sqcup z\}=\min\{z\in\mathfrak{A}\mid b\equiv z\}=b^{+}$. □

Theorem 2.126. $(a\sqcap b)^{+}=a^{+}\sqcup b^{+}$ for every elements a, b of a co-Heyting algebra.

Proof. $a\sqcup(a\sqcap b)^{+}\sqsupseteq(a\sqcap b)\sqcup(a\sqcap b)^{+}\sqsupseteq 1$. So $a\sqcup(a\sqcap b)^{+}\sqsupseteq 1$; $(a\sqcap b)^{+}\sqsupseteq 1\setminus^{*}a=a^{+}$.

We have $(a\sqcap b)^{+}\sqsupseteq a^{+}$. Similarly $(a\sqcap b)^{+}\sqsupseteq b^{+}$. Thus $(a\sqcap b)^{+}\sqsupseteq a^{+}\sqcup b^{+}$.

On the other hand, $a^{+}\sqcup b^{+}\sqcup(a\sqcap b)=(a^{+}\sqcup b^{+}\sqcup a)\sqcap(a^{+}\sqcup b^{+}\sqcup b)$. Obviously $a^{+}\sqcup b^{+}\sqcup a=a^{+}\sqcup b^{+}\sqcup b=1$. So $a^{+}\sqcup b^{+}\sqcup(a\sqcap b)\sqsupseteq 1$ and thus $a^{+}\sqcup b^{+}\sqsupseteq 1\setminus^{*}(a\sqcap b)=(a\sqcap b)^{+}$.

So $(a\sqcap b)^{+}=a^{+}\sqcup b^{+}$. □

2.2 Intro to category theory

I recall that this is a *very* basic introduction to category theory, I even do not define *functors* as they have no use in my theory.

Definition 2.127. A *directed multigraph* is:

1. a set $\mathcal{O}$ (*vertices*);
2. a set $\mathcal{M}$ (*edges*);
3. functions Src and Dst (*source* and *destination*) from $\mathcal{M}$ to $\mathcal{O}$.

Note that in category theory vertices are called *objects* and edges are called *morphisms*.

Definition 2.128. A *precategory* is a directed multigraph together with a partial binary operation $\circ$ on the set $\mathcal{M}$ such that $g\circ f$ is defined iff $\operatorname{Dst} f=\operatorname{Src} g$ (for every morphisms f and g) such that

1. $\operatorname{Src}(g\circ f)=\operatorname{Src} f$ and $\operatorname{Dst}(g\circ f)=\operatorname{Dst} g$ whenever the composition $g\circ f$ of morphisms f and g is defined.
2. $(h\circ g)\circ f=h\circ(g\circ f)$ whenever compositions in this equation are defined.

Definition 2.129. The set $\operatorname{Mor}(A;B)$ (morphisms from an object A to an object B) is exactly morphisms which have A as the source and B as the destination.

Definition 2.130. *Identity morphism* is such a morphism e that $e\circ f=f$ and $g\circ e=g$ whenever compositions in these formulas are defined.

Definition 2.131. A *category* is a precategory with additional requirement that for every object X there exists identity morphism 1_X.

Proposition 2.132. For every object X there exist no more than one identity morphism.

Proof. Let p and q be both identity morphisms for a object X. Then $p=p\circ q=q$. □

Definition 2.133. An *isomorphism* is such a morphism f of a category that there exists a morphism f^{-1} (*inverse* of f) such that $f\circ f^{-1}=1_{\operatorname{Dst} f}$ and $f^{-1}\circ f=1_{\operatorname{Src} f}$.

Proposition 2.134. An isomorphism has exactly one inverse.

Proof. Let g and h be both inverses of f. Then $h=h\circ 1_{\operatorname{Dst} f}=h\circ f\circ g=1_{\operatorname{Src} f}\circ g=g$. □

Definition 2.135. A *groupoid* is a category all of whose morphisms are isomorphisms.

Some important examples of categories:

Exercise 2.3. Prove that the below examples of categories are really categories.

Definition 2.136. The category Set is:

- Objects are small sets.