So $\min \mathfrak{A}$ exists. □

Definition 2.114. *Co-Heyting lattice* is co-brouwerian lattice with greatest element.

Theorem 2.115. For a co-brouwerian lattice $a \sqcup -$ is an upper adjoint of $- \setminus^* a$ for every $a \in \mathfrak{A}$.

Proof. $g(b) = \min \{x \in \mathfrak{A} \mid a \sqcup x \sqsupseteq b\} = b \setminus^* a$ exists for every $b \in \mathfrak{A}$ and thus is the lower adjoint of $a \sqcup -$. □

Corollary 2.116. $\forall a, x, y \in \mathfrak{A}: (x \setminus^* a \sqsubseteq y \Leftrightarrow x \sqsubseteq a \sqcup y)$ for a co-brouwerian lattice.

Definition 2.117. Let $a, b \in \mathfrak{A}$ where $\mathfrak{A}$ is a complete lattice. *Quasidifference* $a \setminus^* b$ is defined by the formula:

$$a \setminus^* b = \bigsqcap \{z \in \mathfrak{A} \mid a \sqsubseteq b \sqcup z\}.$$

Remark 2.118. A more detailed theory of quasidifference (as well as quasicomplement and dual quasicomplement) will be considered below.

Lemma 2.119. $(a \setminus^* b) \sqcup b = a \sqcup b$ for elements a, b of a meet infinite distributive complete lattice.

Proof.

$$\begin{aligned}(a \setminus^* b) \sqcup b &= \\ \bigsqcap \{z \in \mathfrak{A} \mid a \sqsubseteq b \sqcup z\} \sqcup b &= \\ \bigsqcap \{z \sqcup b \mid z \in \mathfrak{A}, a \sqsubseteq b \sqcup z\} &= \\ \bigsqcap \{t \in \mathfrak{A} \mid t \sqsupseteq b, a \sqsubseteq t\} &= \\ a \sqcup b. &\end{aligned}$$

□

Theorem 2.120. The following are equivalent for a complete lattice $\mathfrak{A}$:

1. $\mathfrak{A}$ is meet infinite distributive.
2. $\mathfrak{A}$ is a co-brouwerian lattice.
3. $\mathfrak{A}$ is a co-Heyting lattice.
4. $a \sqcup -$ has lower adjoint for every $a \in \mathfrak{A}$.

Proof.

(2)⇔(3). Obvious (taking into account completeness of $\mathfrak{A}$).

(4)⇒(1). Let $- \setminus^* a$ be the lower adjoint of $a \sqcup -$. Let $S \in \mathscr{P}\mathfrak{A}$. For every $y \in S$ we have $y \sqsupseteq (a \sqcup y) \setminus^* a$ by properties of Galois connections; consequently $y \sqsupseteq (\bigsqcap \langle a \sqcup \rangle S) \setminus^* a$; $\bigsqcap S \sqsupseteq (\bigsqcap \langle a \sqcup \rangle S) \setminus^* a$. So

$$a \sqcup \bigsqcap S \sqsupseteq \left(\left(\bigsqcap \langle a \sqcup \rangle S\right) \setminus^* a\right) \sqcup a \sqsupseteq \bigsqcap \langle a \sqcup \rangle S.$$

But $a \sqcup \bigsqcap S \sqsubseteq \bigsqcap \langle a \sqcup \rangle S$ is obvious.

(1)⇒(2). Let $a \setminus^* b = \bigsqcap \{z \in \mathfrak{A} \mid a \sqsubseteq b \sqcup z\}$. To prove that $\mathfrak{A}$ is a co-brouwerian lattice it is enough to prove $a \sqsubseteq b \sqcup (a \setminus^* b)$. But it follows from the lemma.

(2)⇒(4). $a \setminus^* b = \min \{z \in \mathfrak{A} \mid a \sqsubseteq b \sqcup z\}$. So $a \sqcup -$ is the upper adjoint of $- \setminus^* a$.

(1)⇒(4). Because $a \sqcup -$ preserves all meets. □

Corollary 2.121. Co-brouwerian lattices are distributive.

The following theorem is essentially borrowed from [18]: