Obvious 2.13. Dual of a partial order is a partial order.

Definition 2.14. The *dual* poset for a poset $(A; \sqsubseteq)$ is the poset $(A; \sqsupseteq)$.

Below we will sometimes use *duality* that is replacement of the partial order and all related operations and relations with their duals. In other words, it is enough to prove a theorem for an order $\sqsubseteq$ and the similar theorem for $\sqsupseteq$ follows by duality.

2.1.1.1 Intersecting and joining elements

Let $\mathfrak{A}$ be a poset.

Definition 2.15. Call elements a and b of $\mathfrak{A}$ *intersecting*, denoted $a \not\asymp b$, when there exists a non-least element c such that $c \sqsubseteq a \wedge c \sqsubseteq b$.

Definition 2.16. $a \asymp b \overset{\text{def}}{=} \neg(a \not\asymp b)$.

Obvious 2.17. $a_0 \not\asymp b_0 \wedge a_1 \sqsupseteq a_0 \wedge b_1 \sqsupseteq b_0 \Rightarrow a_1 \not\asymp b_1$.

Definition 2.18. I call elements a and b of $\mathfrak{A}$ *joining* and denote $a \equiv b$ when there is no a non-greatest element c such that $c \sqsupseteq a \wedge c \sqsupseteq b$.

Definition 2.19. $a \not\equiv b \overset{\text{def}}{=} \neg(a \equiv b)$.

Obvious 2.20. Intersecting is the dual of non-joining.

Obvious 2.21. $a_0 \equiv b_0 \wedge a_1 \sqsupseteq a_0 \wedge b_1 \sqsupseteq b_0 \Rightarrow a_1 \equiv b_1$.

2.1.2 Linear order

Definition 2.22. A poset $\mathfrak{A}$ is called *linearly ordered set* (or what is the same, *totally ordered set*) if $a \sqsupseteq b \vee b \sqsupseteq a$ for every $a, b \in \mathfrak{A}$.

Example 2.23. The set of real numbers with the customary order is a linearly ordered set.

Definition 2.24. A set $X \in \mathscr{P}\mathfrak{A}$ where $\mathfrak{A}$ is a poset is called a *chain* if $\mathfrak{A}$ restricted to X is a total order.

2.1.3 Meets and joins

Let $\mathfrak{A}$ be a poset.

Definition 2.25. Given a set $X \in \mathscr{P}\mathfrak{A}$ the *least element* (also called *minimum* and denoted $\min X$) of X is such $a \in X$ that $\forall x \in X: a \sqsubseteq x$.

Least element does not necessarily exists. But if it exists:

Proposition 2.26. For a given $X \in \mathscr{P}\mathfrak{A}$ there exist no more than one least element.

Proof. It follows from anti-symmetry. □

Greatest element is the dual of least element:

Definition 2.27. Given a set $X \in \mathscr{P}\mathfrak{A}$ the *greatest element* (also called *maximum* and denoted $\max X$) of X is such $a \in X$ that $\forall x \in X: a \sqsupseteq x$.

Remark 2.28. Least and greatest elements of a set X is a trivial generalization of the above defined least and greatest element for the entire poset.