Chapter 2
Common knowledge, part 1

In this chapter we will consider some well known mathematical theories. If you already know them you may skip reading this chapter (or its parts).

2.1 Order theory

2.1.1 Posets

Definition 2.1. The *identity relation* on a set A is $\mathrm{id}_A = \{(a; a) \mid a \in A\}$.

Definition 2.2. A *preorder* on a set A is a binary relation $\sqsubseteq$ which is:

- reflexive on A ($(\sqsubseteq) \supseteq \mathrm{id}_A$);
- transitive ($(\sqsubseteq) \circ (\sqsubseteq) \subseteq (\sqsubseteq)$).

Definition 2.3. A *partial order* on a set A is a preorder on A which is antisymmetric ($(\sqsubseteq) \cap (\sqsubseteq)^{-1} \subseteq (=)$).

The reverse relation is denoted $\sqsupseteq$.

Definition 2.4. a is a *subelement* of b (or what is the same a is *contained* in b or b *contains* a) iff $a \sqsubseteq b$.

Obvious 2.5. The reverse of a partial order is also a partial order.

Definition 2.6. A poset is a set A together with a partial order on it is called a *partially ordered set* (*poset* for short).

Definition 2.7. *Strict partial order* $\sqsubset$ corresponding to the partial order $\sqsubseteq$ on a set A is defined by the formula $(\sqsubset) = (\sqsubseteq) \setminus \mathrm{id}_A$.

Definition 2.8. A partial order on a set A *restricted* to a set $B \subseteq A$ is $(\sqsubseteq) \cap (B \times B)$.

Obvious 2.9. A partial order on a set A restricted to a set $B \subseteq A$ is a partial order on B.

Definition 2.10.

- The *least* element 0 of a poset $\mathfrak{A}$ is defined by the formula $\forall a \in \mathfrak{A}: 0 \sqsubseteq a$.
- The *greatest* element 1 of a poset $\mathfrak{A}$ is defined by the formula $\forall a \in \mathfrak{A}: 1 \sqsupseteq a$.

Proposition 2.11. There exist no more than one least element and no more than one greatest element (for a given poset).

Proof. By antisymmetry. □

Definition 2.12. The *dual* order for $\sqsubseteq$ is $\sqsupseteq$.