QUESTION 1519. Is $\mathrm{ID}^{\mathsf{Strd}}_{A[n]}$ principal for every principal filter A on a set and index set n?

PROPOSITION 1520. $\uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \sqsubseteq \downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{\uparrow A[n]}$ for every set A.

PROOF.

$$L \in \mathrm{GR} \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \Leftrightarrow L \in \mathrm{GR}\, \mathrm{id}^{\mathsf{Strd}}_{\uparrow A[n]} \Leftrightarrow \uparrow A \not\asymp \bigsqcap^{\mathfrak{A}}_{i\in n} L_i \Leftarrow$$
$$\uparrow A \not\asymp \bigsqcap^{\mathfrak{Z}}_{i\in n} L_i \Leftrightarrow L \in \downdownarrows \mathrm{GR}\, \mathrm{ID}^{\mathsf{Strd}}_{\uparrow A[n]}.$$

□

PROPOSITION 1521. $\uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \sqsubset \downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{\uparrow A[n]}$ for some set A and index set n.

PROOF. $L \in \mathrm{GR} \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \Leftrightarrow \bigsqcap^{\mathfrak{Z}}_{i\in n} L_i \not\asymp \uparrow A$ what is not implied by $\bigsqcap^{\mathfrak{A}}_{i\in n} L_i \not\asymp \uparrow A$ that is $L \in \downdownarrows \mathrm{GR}\, \mathrm{ID}^{\mathsf{Strd}}_{\uparrow A[n]}$. (For a counter example take $n = \mathbb{N}$, $L_i = (0; 1/i)$, $A = \mathbb{R}$.) □

PROPOSITION 1522. $\upuparrows \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} = \upuparrows \mathrm{id}^{\mathsf{Strd}}_{\uparrow A[n]}$.

PROOF. $\upuparrows \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} = \upuparrows \mathrm{id}^{\mathsf{Strd}}_{\uparrow A[n]}$ is obvious from the above. □

PROPOSITION 1523. $\upuparrows \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \sqsubseteq \mathrm{ID}^{\mathsf{Strd}}_{\uparrow A[n]}$.

PROOF.

$$\mathcal{X} \in \mathrm{GR} \upuparrows \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \Leftrightarrow \mathrm{up}\, \mathcal{X} \subseteq \mathrm{GR} \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \Leftrightarrow$$
$$\forall Y \in \mathrm{up}\, \mathcal{X} : Y \in \mathrm{GR} \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \Leftrightarrow \forall Y \in \mathrm{up}\, \mathcal{X} : Y \in \mathrm{id}^{\mathsf{Strd}}_{\uparrow A[n]} \Leftrightarrow$$
$$\forall Y \in \mathrm{up}\, \mathcal{X} : \bigsqcap^{\mathfrak{Z}}_{i\in n} Y_i \sqcap \uparrow A \neq \bot \Rightarrow \bigsqcap^{\mathfrak{A}}_{i\in n} \mathcal{X}_i \sqcap \uparrow A \neq 0 \Leftrightarrow \mathcal{X} \in \mathrm{GR}\, \mathrm{ID}^{\mathsf{Strd}}_{\uparrow A[n]}.$$

□

PROPOSITION 1524. $\upuparrows \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \sqsubset \mathrm{ID}^{\mathsf{Strd}}_{\uparrow A[n]}$ for some set A.

PROOF. We need to prove $\upuparrows \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \neq \mathrm{ID}^{\mathsf{Strd}}_{\uparrow A[n]}$ that is it's enough to prove (see the above proof) that $\forall Y \in \mathrm{up}\, \mathcal{X} : \bigsqcap^{\mathfrak{Z}}_{i\in n} Y_i \sqcap \uparrow A \neq \bot \not\Leftarrow \bigsqcap^{\mathfrak{A}}_{i\in n} \mathcal{X}_i \sqcap \uparrow A \neq \bot$. A counter-example follows:

$\forall Y \in \mathrm{up}\, \mathcal{X} : \bigsqcap^{\mathfrak{Z}}_{i\in n} Y_i \sqcap \uparrow A \neq \bot$ does not hold for $n = \mathbb{N}$, $\mathcal{X}_i = \uparrow (-1/i; 0)$ for $i \in n$, $A = (-\infty; 0)$. To show this, it's enough to prove $\bigsqcap^{\mathfrak{Z}}_{i\in n} Y_i \sqcap \uparrow A \neq \bot$ for $Y_i = \uparrow (-1/i; 0)$ but this is obvious since $\bigsqcap^{\mathfrak{Z}}_{i\in n} Y_i = \bot$.

On the other hand, $\bigsqcap^{\mathfrak{A}}_{i\in n} \mathcal{X}_i \sqcap \uparrow A \neq \bot$ for the same $\mathcal{X}$ and A. □

The above theorems are summarized in the diagram at figure 1:

$$\begin{array}{ccc}
\downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{\uparrow A[n]} & \sqsupseteq & \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} = \mathrm{id}^{\mathsf{Strd}}_{\uparrow A[n]} \\
\downarrow \upuparrows \quad \uparrow \downdownarrows & & \downarrow \upuparrows \quad \uparrow \downdownarrows \\
\mathrm{ID}^{\mathsf{Strd}}_{\uparrow A[n]} & \sqsupseteq & \upuparrows \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} = \upuparrows \mathrm{id}^{\mathsf{Strd}}_{\uparrow A[n]}
\end{array}$$

FIGURE 1. Relationships of identity staroids for principal filters.