COROLLARY 1509. Staroidal product of an infinite indexed family of ultrafilters may be non-atomic.

PROPOSITION 1510. $\mathrm{id}^{\mathsf{Strd}}_{a[n]}$ is determined by the value of $\upuparrows \mathrm{id}^{\mathsf{Strd}}_{a[n]}$. Moreover $\mathrm{id}^{\mathsf{Strd}}_{a[n]} = \downdownarrows\upuparrows \mathrm{id}^{\mathsf{Strd}}_{a[n]}$.

PROOF. Use general properties of upgrading and downgrading (proposition 1261). □

LEMMA 1511. $\mathcal{L} \in \mathrm{GR}\,\mathrm{ID}^{\mathsf{Strd}}_{a[n]}$ iff $\bigcup_{i\in n} \mathcal{L}_i \cup a$ has finite intersection property (for primary filtrators).

PROOF. $\mathcal{L} \in \mathrm{GR}\,\mathrm{ID}^{\mathsf{Strd}}_{a[n]} \Leftrightarrow \bigsqcap_{i\in n} \mathcal{L} \sqcap a \neq 0^{\mathfrak{F}} \Leftrightarrow \forall X \in \bigsqcap_{i\in n} \mathcal{L} \sqcap a : X \neq \emptyset$ what is equivalent of $\bigcup_{i\in n} \mathcal{L}_i \cup a$ having finite intersection property. □

PROPOSITION 1512. $\mathrm{ID}^{\mathsf{Strd}}_{a[n]}$ is determined by the value of $\downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{a[n]}$, moreover $\mathrm{ID}^{\mathsf{Strd}}_{a[n]} = \upuparrows\downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{a[n]}$ (for primary filtrators).

PROOF.

$$\mathcal{L} \in \upuparrows\downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{a[n]} \Leftrightarrow \mathrm{up}\,\mathcal{L} \subseteq \downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{a[n]} \Leftrightarrow \mathrm{up}\,\mathcal{L} \subseteq \mathrm{ID}^{\mathsf{Strd}}_{a[n]} \Leftrightarrow$$
$$\forall L \in \mathrm{up}\,\mathcal{L} : L \in \mathrm{ID}^{\mathsf{Strd}}_{a[n]} \Leftrightarrow \forall L \in \mathrm{up}\,\mathcal{L} : \bigsqcap_{i\in n} L_i \sqcap a \neq 0^{\mathfrak{F}} \Leftrightarrow$$
$$\bigcup_{i\in n} \mathcal{L}_i \cup a \text{ has finite intersection property} \Leftrightarrow (\text{lemma}) \Leftrightarrow \mathcal{L} \in \mathrm{GR}\,\mathrm{ID}^{\mathsf{Strd}}_{a[n]}.$$

□

PROPOSITION 1513. $\mathrm{id}^{\mathsf{Strd}}_{a[n]} \sqsubseteq \downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{a[n]}$ for every filter a and an index set n.

PROOF. $\mathrm{id}^{\mathsf{Strd}}_{a[n]} = \downdownarrows\upuparrows \mathrm{id}^{\mathsf{Strd}}_{a[n]} \sqsubseteq \downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{a[n]}$. □

PROPOSITION 1514. $\mathrm{id}^{\mathsf{Strd}}_{a[a]} \sqsubset \downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{a[a]}$ for every nontrivial ultrafilter a.

PROOF. Suppose $\mathrm{id}^{\mathsf{Strd}}_{a[a]} = \downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{a[a]}$. Then $\mathrm{ID}^{\mathsf{Strd}}_{a[a]} = \upuparrows\downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{a[a]} = \upuparrows \mathrm{id}^{\mathsf{Strd}}_{a[a]}$ what contradicts to the above. □

OBVIOUS 1515. $\mathcal{L} \in \mathrm{GR}\,\mathrm{ID}^{\mathsf{Strd}}_{a[n]} \Leftrightarrow a \sqcap \bigsqcap_{i\in n} \mathcal{L}_i \neq \bot^{\mathfrak{F}}$ if a is an element of a complete lattice.

OBVIOUS 1516. $\mathcal{L} \in \mathrm{GR}\,\mathrm{ID}^{\mathsf{Strd}}_{a[n]} \Leftrightarrow \forall i \in n : \mathcal{L}_i \sqsupseteq a \Leftrightarrow \forall i \in n : \mathcal{L}_i \not\asymp a$ if a is an ultrafilter on $\mathfrak{A}$.

18.4.6. Identity staroids on principal filters. For principal filter $\uparrow A$ (where A is a set) the above definitions coincide with n-ary identity relation, as formulated in the following propositions:

PROPOSITION 1517. $\uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} = \mathrm{id}^{\mathsf{Strd}}_{\uparrow A[n]}$.

PROOF.

$$L \in \mathrm{GR} \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \Leftrightarrow \prod L \not\asymp \mathrm{id}_{A[n]} \Leftrightarrow \exists t \in A \forall i \in n : t \in L_i \Leftrightarrow$$
$$\bigcap_{i\in n} L_i \cap A \neq \emptyset \Leftrightarrow L \in \mathrm{GR}\,\mathrm{id}^{\mathsf{Strd}}_{\uparrow A[n]}.$$

Thus $\uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} = \mathrm{id}^{\mathsf{Strd}}_{\uparrow A[n]}$. □

COROLLARY 1518. $\mathrm{id}^{\mathsf{Strd}}_{\uparrow A[n]}$ is a principal staroid.