18.3.1.1. *Completely starrish posets.*

DEFINITION 1466. I will call a poset *completely starrish* when the full star $\star a$ is a complete free star for every element a of this poset.

OBVIOUS 1467. Every completely starrish poset is starrish.

PROPOSITION 1468. Every complete join infinite distributive lattice is starrish.

PROOF. Let $\mathfrak{A}$ be a join infinite distributive lattice, $a \in \mathfrak{A}$. Obviously $\bot \notin \star a$ (if $\bot$ exists); obviously $\star a$ is an upper set. If $\bigsqcup T \in \star a$, then $(\bigsqcup T) \sqcap a$ is non-least that is $\bigsqcup \langle a \sqcap \rangle^* T$ is non-least what is equivalent to $a \sqcap x$ being non-least for some $x \in T$ that is $x \in \star a$. □

THEOREM 1469. If $\mathfrak{A}$ is a completely starrish complete lattice lattice then

$$\operatorname{atoms} \bigsqcup T = \bigcup \langle \operatorname{atoms} \rangle^* T.$$

for every $T \in \mathscr{P}\mathfrak{A}$.

PROOF. For every atom c we have:

$$c \in \operatorname{atoms} \bigsqcup T \Leftrightarrow c \not\asymp \bigsqcup T \Leftrightarrow \bigsqcup T \in \star c \Leftrightarrow \exists X \in T : X \in \star c \Leftrightarrow$$
$$\exists X \in T : X \not\asymp c \Leftrightarrow \exists X \in T : c \in \operatorname{atoms} X \Leftrightarrow c \in \bigcup \langle \operatorname{atoms} \rangle^* T.$$

□

18.3.2. More on free stars and complete free stars.

OBVIOUS 1470. $\partial \mathcal{F} = \downdownarrows \star \mathcal{F}$ for an element $\mathcal{F}$ of down-aligned finitely meet closed filtrator.

COROLLARY 1471. $\partial \mathcal{F} = \downdownarrows \star \mathcal{F}$ for every filter $\mathcal{F}$ on a poset.

PROPOSITION 1472. $\star \mathcal{F} = \upuparrows \partial \mathcal{F}$ for an element $\mathcal{F}$ of a filtrator with separable core.

PROOF. $\mathcal{X} \in \upuparrows \partial \mathcal{F} \Leftrightarrow \operatorname{up} \mathcal{X} \subseteq \partial \mathcal{F} \Leftrightarrow \forall X \in \operatorname{up} \mathcal{X} : X \not\asymp \mathcal{F} \Leftrightarrow \mathcal{X} \not\asymp \mathcal{F} \Leftrightarrow \mathcal{X} \in \star \mathcal{F}$. □

COROLLARY 1473. $\star \mathcal{F} = \upuparrows \partial \mathcal{F}$ for every filter $\mathcal{F}$ on a distributive lattice with least element.

PROPOSITION 1474. For a semifiltered, star-separable, down-aligned filtrator $(\mathfrak{A}; \mathfrak{Z})$ with finitely meet closed and separable core where $\mathfrak{Z}$ is a complete boolean lattice and both $\mathfrak{Z}$ and $\mathfrak{A}$ are atomistic lattices the following conditions are equivalent for any $\mathcal{F} \in \mathfrak{A}$:

1°. $\mathcal{F} \in \mathfrak{Z}$.
2°. $\partial \mathcal{F}$ is a complete free star on $\mathfrak{Z}$.
3°. $\star \mathcal{F}$ is a complete free star on $\mathfrak{F}$.

PROOF.

1°⇒2°. That $\partial \mathcal{F}$ does not contain the least element is obvious. That $\partial \mathcal{F}$ is an upper set is obvious. So it remains to apply theorem 320.
2°⇒3°. That $\star \mathcal{F}$ does not contain the least element is obvious. That $\star \mathcal{F}$ is an upper set is obvious. So it remains to apply theorem 320.
3°⇒1°. Apply theorem 320.

□

COROLLARY 1475. For a filter $\mathcal{F} \in \mathfrak{F}$ on a complete atomic boolean lattice the following conditions are equivalent: