Otherwise $(\operatorname{val}\vartheta)_i L = \emptyset$. Thus $(\operatorname{val}\vartheta)_i L$ is a free star. So ϑ is a staroid.FiXme: Show that it's not just a prestaroid. □

PROPOSITION 1426. ϑ is a completary staroid.

PROOF.

$$A_0 \sqcup A_1 \in \operatorname{GR}\vartheta \Leftrightarrow A_0 \cup A_1 \in \operatorname{GR}\vartheta \Leftrightarrow \sup_{i\in\mathbb{N}} \operatorname{card}((A_0 i \cup A_1 i) \cap i) = \mathbb{N} \wedge \forall i \in \mathbb{N} : A_0 i \cup A_1 i \neq \emptyset \Leftrightarrow \sup_{i\in\mathbb{N}} \operatorname{card}((A_0 i \cap i) \cup (A_1 i \cap i)) = \mathbb{N} \wedge \forall i \in \mathbb{N} : A_0 i \cup A_1 i \neq \emptyset.$$

If $A_0 i = \emptyset$ then $A_0 i \cap i = \emptyset$ and thus $A_1 i \cap i \sqsupseteq A_0 i \cap i$. Thus we can select $c(i) \in \{0, 1\}$ in such a way that $\forall d \in \{0, 1\} : \operatorname{card}(A_{c(i)} i \cap i) \sqsupseteq \operatorname{card}(A_d i \cap i)$ and $A_{c(i)} i \neq \emptyset$. (Consider the case $A_0 i, A_1 i \neq \emptyset$ and the similar cases $A_0 i = \emptyset$ and $A_1 i = \emptyset$.)

So

$$A_0 \sqcup A_1 \in \operatorname{GR}\vartheta \Leftrightarrow \sup_{i\in\mathbb{N}} \operatorname{card}(A_{c(i)} i \cap i) = \mathbb{N} \wedge \forall i \in \mathbb{N} : A_{c(i)} i \neq \emptyset \Leftrightarrow (\lambda i \in n : A_{c(i)} i) \in \operatorname{GR}\vartheta.$$

Thus ϑ is completary. □

OBVIOUS 1427. ϑ is non-zero.

EXAMPLE 1428. For every family $a = a_{i\in\mathbb{N}}$ of ultrafilters $\prod^{\mathsf{Strd}} a$ is not an atom nor of the poset of staroids neither of the poset of completary staroids of the form $\lambda i \in \mathbb{N} : \operatorname{Base}(a_i)$.

PROOF. It's enough to prove $\vartheta \not\sqsupseteq \prod^{\mathsf{Strd}} a$.
Let $\uparrow^{\mathbb{N}} R_i = a_i$ if a_i is principal and $R_i = \mathbb{N} \setminus i$ if a_i is non-principal.
We have $\forall i \in \mathbb{N} : R_i \in a_i$.
We have $R \notin \operatorname{GR}\vartheta$ because $\sup_{i\in\mathbb{N}} \operatorname{card}(R_i \cap i) \neq \mathbb{N}$.
$R \in \prod^{\mathsf{Strd}} a$ because $\forall X \in a_i : X \cap R_i \neq \emptyset$.
So $\vartheta \not\sqsupseteq \prod^{\mathsf{Strd}} a$. □

REMARK 1429. At http://mathoverflow.net/questions/60925/special-infinitary-relations-and-ultrafilters there are a proof for arbitrary infinite form, not just for $\mathbb{N}$.

CONJECTURE 1430. There exists a non-completary staroid.

CONJECTURE 1431. There exists a prestaroid which is not a staroid.

CONJECTURE 1432. The set of staroids of the form A^B where A and B are sets is atomic.

CONJECTURE 1433. The set of staroids of the form A^B where A and B are sets is atomistic.

CONJECTURE 1434. The set of completary staroids of the form A^B where A and B are sets is atomic.

CONJECTURE 1435. The set of completary staroids of the form A^B where A and B are sets is atomistic.

EXAMPLE 1436. $\operatorname{StarComp}(a; f \sqcup g) \neq \operatorname{StarComp}(a; f) \sqcup \operatorname{StarComp}(a; g)$ in the category of binary relations with star-morphisms for some n-ary relation a and an n-indexed families f and g of functions.