PROOF.

$$\uparrow^{\mathsf{FCD}} A \left[f \times^{(C)} g \right] \uparrow^{\mathsf{FCD}} B \Leftrightarrow$$

$$\exists a \in \operatorname{atoms} \uparrow^{\mathsf{FCD}} A, b \in \operatorname{atoms} \uparrow^{\mathsf{FCD}} B : a \left[f \times^{(C)} g \right] b \Leftrightarrow$$

$$\exists a \in \operatorname{atoms} \uparrow^{\mathsf{FCD}} A, b \in \operatorname{atoms} \uparrow^{\mathsf{FCD}} B : (\operatorname{dom} a \, [f] \operatorname{dom} b \wedge \operatorname{im} a \, [g] \operatorname{im} b) \Rightarrow$$

$$\exists a_0 \in \operatorname{atoms} \operatorname{dom} \uparrow^{\mathsf{FCD}} A, a_1 \in \operatorname{atoms} \operatorname{im} \uparrow^{\mathsf{FCD}} A,$$

$$b_0 \in \operatorname{atoms} \operatorname{dom} \uparrow^{\mathsf{FCD}} B, b_1 \in \operatorname{atoms} \operatorname{im} \uparrow^{\mathsf{FCD}} B : (a_0 \, [f] \, b_0 \wedge a_1 \, [g] \, b_1).$$

On the other hand:

$$\exists a_0 \in \operatorname{atoms} \operatorname{dom} \uparrow^{\mathsf{FCD}} A, a_1 \in \operatorname{atoms} \operatorname{im} \uparrow^{\mathsf{FCD}} A,$$

$$b_0 \in \operatorname{atoms} \operatorname{dom} \uparrow^{\mathsf{FCD}} B, b_1 \in \operatorname{atoms} \operatorname{im} \uparrow^{\mathsf{FCD}} B : (a_0 \, [f] \, b_0 \wedge a_1 \, [g] \, b_1) \Rightarrow$$

$$\exists a_0 \in \operatorname{atoms} \operatorname{dom} \uparrow^{\mathsf{FCD}} A, a_1 \in \operatorname{atoms} \operatorname{im} \uparrow^{\mathsf{FCD}} A,$$

$$b_0 \in \operatorname{atoms} \operatorname{dom} \uparrow^{\mathsf{FCD}} B, b_1 \in \operatorname{atoms} \operatorname{im} \uparrow^{\mathsf{FCD}} B : (a_0 \times^{\mathsf{FCD}} b_0 \not\asymp f \wedge a_1 \times^{\mathsf{FCD}} b_1 \not\asymp g) \Rightarrow$$

$$\exists a \in \operatorname{atoms} \uparrow^{\mathsf{FCD}} A, b \in \operatorname{atoms} \uparrow^{\mathsf{FCD}} B : (\operatorname{dom} a \, [f] \operatorname{dom} b \wedge \operatorname{im} a \, [g] \operatorname{im} b).$$

Also using the lemma we have

$$\exists a \in \operatorname{atoms} \uparrow^{\mathsf{FCD}} A, b \in \operatorname{atoms} \uparrow^{\mathsf{FCD}} B : (\operatorname{dom} a \, [f] \operatorname{dom} b \wedge \operatorname{im} a \, [g] \operatorname{im} b) \Leftrightarrow$$

$$\exists a \in \operatorname{atoms} \uparrow^{\mathsf{RLD}} A, b \in \operatorname{atoms} \uparrow^{\mathsf{RLD}} B : (\operatorname{dom} a \, [f] \operatorname{dom} b \wedge \operatorname{im} a \, [g] \operatorname{im} b).$$

So

$$\uparrow^{\mathsf{FCD}} A \left[f \times^{(C)} g \right] \uparrow^{\mathsf{FCD}} B \Leftrightarrow$$

$$\exists a \in \operatorname{atoms} \uparrow^{\mathsf{RLD}} A, b \in \operatorname{atoms} \uparrow^{\mathsf{RLD}} B : (\operatorname{dom} a \, [f] \operatorname{dom} b \wedge \operatorname{im} a \, [g] \operatorname{im} b) \Leftrightarrow$$

$$\exists a \in \operatorname{atoms} \uparrow^{\mathsf{RLD}} A, b \in \operatorname{atoms} \uparrow^{\mathsf{RLD}} B : a \left[f \times^{(A)} g \right] b \Leftrightarrow$$

$$\uparrow^{\mathsf{RLD}} A \left[f \times^{(A)} g \right] \uparrow^{\mathsf{RLD}} B.$$

□

COROLLARY 1418. $f \times^{(A)} g = \upuparrows \downdownarrows (f \times^{(C)} g)$ where downgrading is taken on the filtrator

$$\left(\mathsf{FCD}(\mathsf{FCD}(\operatorname{Src} \circ f); \mathsf{FCD}(\operatorname{Dst} \circ f)); \mathsf{FCD}\left(\mathscr{P} \prod (\operatorname{Src} \circ f); \mathscr{P} \prod (\operatorname{Dst} \circ f) \right) \right)$$

and upgrading is taken on the filtrator

$$\left(\mathsf{FCD}(\mathsf{RLD}(\operatorname{Src} \circ f); \mathsf{RLD}(\operatorname{Dst} \circ f)); \mathsf{FCD}\left(\mathscr{P} \prod (\operatorname{Src} \circ f); \mathscr{P} \prod (\operatorname{Dst} \circ f) \right) \right).$$

where we equate n-ary relations with corresponding principal multifuncoids and principal multireloids, when appropriate.

PROOF. Leave as an exercise for the reader. □

CONJECTURE 1419. $\uparrow^{\mathsf{FCD}} A \left[\prod^{(C)} f \right] \uparrow^{\mathsf{FCD}} B \Leftrightarrow \uparrow^{\mathsf{RLD}} A \left[\prod^{(A)} f \right] \uparrow^{\mathsf{RLD}} B$ for every indexed family f of funcoids and $A \in \mathscr{P} \prod_{i \in \operatorname{dom} f} \operatorname{Src} f_i$, $B \in \mathscr{P} \prod_{i \in \operatorname{dom} f} \operatorname{Dst} f_i$.

THEOREM 1420. For every filters a_0, a_1, b_0, b_1 we haveFiXme: It's a corollary of a below conjecture.

$$a_0 \times^{\mathsf{FCD}} b_0 \left[f \times^{(C)} g \right] a_1 \times^{\mathsf{FCD}} b_1 \Leftrightarrow a_0 \times^{\mathsf{RLD}} b_0 \left[f \times^{(A)} g \right] a_1 \times^{\mathsf{RLD}} b_1.$$