Proof.

$$\mathcal{X}\left[\Pr_k^{(A)} f\right]\mathcal{Y}\Leftrightarrow$$

$$\forall X\in\mathcal{X},Y\in\mathcal{Y}: X\left[\Pr_k^{(A)} f\right]^* Y\Leftrightarrow$$

$$\forall X\in\mathcal{X},Y\in\mathcal{Y}:\prod^{\mathsf{RLD}}_{i\in\operatorname{dom}A}\left(\begin{cases}1^{\mathfrak{F}(A_i)} & \text{if } i\neq k;\\ \uparrow^{A_i} X & \text{if } i=k\end{cases}\right)[f]\prod^{\mathsf{RLD}}_{i\in\operatorname{dom}B}\left(\begin{cases}1^{\mathfrak{F}(B_i)} & \text{if } i\neq k;\\ \uparrow^{B_i} Y & \text{if } i=k\end{cases}\right)\Leftrightarrow$$

$$\forall X\in\prod^{\mathsf{RLD}}_{i\in\operatorname{dom}A}\left(\begin{cases}1^{\mathfrak{F}(A_i)} & \text{if } i\neq k;\\ \mathcal{X} & \text{if } i=k\end{cases}\right),Y\in\prod^{\mathsf{RLD}}_{i\in\operatorname{dom}B}\left(\begin{cases}1^{\mathfrak{F}(B_i)} & \text{if } i\neq k;\\ \mathcal{Y} & \text{if } i=k\end{cases}\right): X\,[f]^*\,Y\Leftrightarrow$$

$$\prod^{\mathsf{RLD}}_{i\in\operatorname{dom}A}\left(\begin{cases}\top^{\mathfrak{F}(A_i)} & \text{if } i\neq k;\\ \mathcal{X} & \text{if } i=k\end{cases}\right)[f]\prod^{\mathsf{RLD}}_{i\in\operatorname{dom}B}\left(\begin{cases}\top^{\mathfrak{F}(B_i)} & \text{if } i\neq k;\\ \mathcal{Y} & \text{if } i=k\end{cases}\right).$$

□

Remark 1404. Reloidal product above can be replaced with starred reloidal product, because of finite number of non-maximal multipliers in the products.

Obvious 1405. $\Pr_k^{(A)}\prod^{(A)} x = x_k$ provided that x is an indexed family of non-zero funcoids.

17.14.4. Other.

Conjecture 1406. Values x_i (for every $i\in\operatorname{dom}x$) can be restored from the value of $\prod^{(C)} x$ provided that x is an indexed family of non-zero reloids.

Definition 1407. *Displaced product* $\prod^{(DP)} f = \downdownarrows\prod^{(C)} f$ for every indexed family of pointfree funcoids, where downgrading is defined for the filtrator

$$\left(\mathsf{FCD}(\operatorname{StarMor}(\operatorname{Src}\circ f);\operatorname{StarMor}(\operatorname{Dst}\circ f));\langle\uparrow^{\mathsf{FCD}}\rangle^*\mathscr{P}\left(\prod(\operatorname{Src}\circ f)\times\prod(\operatorname{Dst}\circ f)\right)\right).$$

Remark 1408. Displaced product is a funcoid (not just a pointfree funcoid).

Conjecture 1409. Values x_i (for every $i\in\operatorname{dom}x$) can be restored from the value of $\prod^{(DP)} x$ provided that x is an indexed family of non-zero funcoids.

Definition 1410. Let $f\in\mathscr{P}\left(Z^{\coprod Y}\right)$ where Z is a set and Y is a function.

$$\Pr_k^{(D)} f = \Pr_k\left\{\frac{\operatorname{curry} z}{z\in f}\right\}.$$

Proposition 1411. $\Pr_k^{(D)}\prod^{(D)} F = F_k$ for every indexed family F of non-empty relations.

Proof. Obvious. □

Corollary 1412. $\operatorname{GR}\Pr_k^{(D)}\prod^{(D)} F = \operatorname{GR} F_k$ and $\operatorname{form}\Pr_k^{(D)}\prod^{(D)} F = \operatorname{form} F_k$ for every indexed family F of non-empty anchored relations.

17.15. Relationships between cross-composition and subatomic products

Proposition 1413. $a\left[f\times^{(C)} g\right] b\Leftrightarrow\operatorname{dom}a\,[f]\,\operatorname{dom}b\wedge\operatorname{im}a\,[g]\,\operatorname{im}b$ for funcoids f and g and atomic funcoids $a\in\mathsf{FCD}(\operatorname{Src}f;\operatorname{Src}g)$ and $b\in\mathsf{FCD}(\operatorname{Dst}f;\operatorname{Dst}g)$.