

$$\begin{aligned}
& \prod_{\text{RLD}} a \left[\prod_{(A)} f \right] \prod_{\text{RLD}} b \Leftrightarrow \\
& \exists x \in \text{atoms} \prod_{\text{RLD}} a, y \in \text{atoms} \prod_{\text{RLD}} b : x \left[\prod_{(A)} f \right] y \Leftrightarrow \\
& \exists x \in \text{atoms} \prod_{\text{RLD}} a, y \in \text{atoms} \prod_{\text{RLD}} b \forall i \in \text{dom } f : \langle \text{Pr}_i \rangle^* x [f_i] \langle \text{Pr}_i \rangle^* y \Leftrightarrow \\
& \forall i \in \text{dom } f \exists x \in \text{atoms } a_i, y \in \text{atoms } b_i : x [f_i] y \Leftrightarrow \\
& \forall i \in \text{dom } f : a_i [f_i] b_i.
\end{aligned}$$

□

THEOREM 1390. $\langle \prod_{(A)} f \rangle x = \prod_{i \in \text{dom } f}^{\text{RLD}} \langle f_i \rangle \text{Pr}_i^{\text{RLD}} x$ for an indexed family f of funcoids and $x \in \text{atoms}^{\text{RLD}(\lambda i \in \text{dom } f : \text{Src } f_i)}$ for every $n \in \text{dom } f$.

PROOF. For every ultrafilter $y \in \mathfrak{F}(\prod_{i \in \text{dom } f} \text{Dst } f_i)$ we have:

$$\begin{aligned}
y \not\prec \prod_{i \in \text{dom } f}^{\text{RLD}} \langle f_i \rangle \text{Pr}_i^{\text{RLD}} x &\Leftrightarrow \\
\forall i \in \text{dom } f : \text{Pr}_i^{\text{RLD}} y \not\prec \langle f_i \rangle \text{Pr}_i^{\text{RLD}} x &\Leftrightarrow \\
\forall i \in \text{dom } f : \text{Pr}_i^{\text{RLD}} x [f_i] \text{Pr}_i^{\text{RLD}} y &\Leftrightarrow \\
x \left[\prod_{(A)} f \right] y &\Leftrightarrow \\
y \not\prec \left\langle \prod_{(A)} f \right\rangle x. &
\end{aligned}$$

Thus $\langle \prod_{(A)} f \rangle x = \prod_{i \in \text{dom } f}^{\text{RLD}} \langle f_i \rangle \text{Pr}_i^{\text{RLD}} x$. □

COROLLARY 1391. $\langle f \times^{(A)} g \rangle x = \langle f \rangle(\text{dom } x) \times^{\text{RLD}} \langle g \rangle(\text{im } x)$.

17.14. On products and projections

CONJECTURE 1392. For principal funcoids $\prod^{(C)}$ and $\prod^{(A)}$ coincide with the conventional product of binary relations.

17.14.1. Staroidal product. Let f be a staroid, whose form components are boolean lattices.

DEFINITION 1393. *Staroidal projection* of a staroid f is the filter $\text{Pr}_k^{\text{Strd}} f$ corresponding to the free star

$$(\text{val } f)_k(\lambda i \in (\text{arity } f) \setminus \{k\} : \top^{(\text{form } f)_i}).$$

PROPOSITION 1394. $\text{Pr}_k \text{GR} \prod^{\text{Strd}} x = \star x_k$ if x is an indexed family of proper filters, and $k \in \text{dom } x$.