Proof. FiXme: Is there necessity to consider $S=\emptyset$ special case?If $S=\emptyset$ then $\bigsqcap\left\{\frac{\prod^{\mathsf{RLD}} a}{a\in S}\right\} = \bigsqcap\emptyset = \top^{\mathsf{RLD}(\mathfrak{A})}$ and

$$\prod^{\mathsf{RLD}}_{i\in\operatorname{dom}\mathfrak{A}} \bigsqcap\left\langle\uparrow^{\mathfrak{F}(\mathfrak{A}_i)}\right\rangle^* \Pr_i S = \prod^{\mathsf{RLD}}_{i\in\operatorname{dom}\mathfrak{A}} \bigsqcap\left\langle\uparrow^{\mathfrak{F}(\mathfrak{A}_i)}\right\rangle^* \emptyset = \prod^{\mathsf{RLD}}_{i\in\operatorname{dom}\mathfrak{A}} \bigsqcap\emptyset = \prod^{\mathsf{RLD}}_{i\in\operatorname{dom}\mathfrak{A}} \top^{\mathfrak{F}(\mathfrak{A}_i)} = \top^{\mathsf{RLD}(\mathfrak{A})},$$

thus $\bigsqcap\left\{\frac{\prod^{\mathsf{RLD}} a}{a\in S}\right\} = \prod^{\mathsf{RLD}}_{i\in\operatorname{dom}\mathfrak{A}} \bigsqcap\left\langle\uparrow^{\mathfrak{F}(\mathfrak{A}_i)}\right\rangle^* \Pr_i S$.

Let $S\neq\emptyset$.

$\bigsqcap\left\langle\uparrow^{\mathfrak{F}(\mathfrak{A}_i)}\right\rangle^* \Pr_i S \sqsubseteq \bigsqcap\left\langle\uparrow^{\mathfrak{F}(\mathfrak{A}_i)}\right\rangle^* \{a_i\} = a_i$ for every $a\in S$ because $a_i\in\Pr_i S$. Thus $\prod^{\mathsf{RLD}}_{i\in\operatorname{dom}\mathfrak{A}} \bigsqcap\left\langle\uparrow^{\mathfrak{F}(\mathfrak{A}_i)}\right\rangle^* \Pr_i S \sqsubseteq \prod^{\mathsf{RLD}} a$;

$$\bigsqcap\left\{\frac{\prod^{\mathsf{RLD}} a}{a\in S}\right\} \sqsupseteq \prod^{\mathsf{RLD}}_{i\in\operatorname{dom}\mathfrak{A}} \bigsqcap\left\langle\uparrow^{\mathfrak{F}(\mathfrak{A}_i)}\right\rangle^* \Pr_i S.$$

Now suppose $F\in\operatorname{GR}\prod^{\mathsf{RLD}}_{i\in\operatorname{dom}\mathfrak{A}} \bigsqcap\left\langle\uparrow^{\mathfrak{F}(\mathfrak{A}_i)}\right\rangle^* \Pr_i S$. Then there exists $X\in\prod_{i\in\operatorname{dom}\mathfrak{A}} \bigsqcap\left\langle\uparrow^{\mathfrak{F}(\mathfrak{A}_i)}\right\rangle^* \Pr_i S$ such that $F\sqsupseteq\prod X$. It is enough to prove that there exist $a\in S$ such that $F\in\operatorname{GR}\prod^{\mathsf{RLD}} a$. For this it is enough $\prod X\in\operatorname{GR}\prod^{\mathsf{RLD}} a$.

Really, $X_i\in\bigsqcap\left\langle\uparrow^{\mathfrak{F}(\mathfrak{A}_i)}\right\rangle^* \Pr_i S$ thus $X_i\in a_i$ for every $A\in S$ because $\Pr_i S\supseteq\{a_i\}$.

Thus $\prod X\in\operatorname{GR}\prod^{\mathsf{RLD}} a$. □

Definition 1372. I call a multireloid *principal* iff its graph is a principal filter.FiXme: Prove that principal multireloids are the same as multireloid corresponding to a relation.

Definition 1373. I call a multireloid *convex* iff it is a join of reloidal products.

Theorem 1374. $\operatorname{StarComp}(a\sqcup b; f) = \operatorname{StarComp}(a; f)\sqcup\operatorname{StarComp}(b; f)$ for multireloids a, b and an indexed family f of reloids with $\operatorname{Src} f_i = (\operatorname{form} a)_i = (\operatorname{form} b)_i$.

Proof.

$$\begin{aligned}
&\operatorname{GR}(\operatorname{StarComp}(a; f)\sqcup\operatorname{StarComp}(b; f)) = \\
&\bigsqcap\left\{\frac{\uparrow^{\mathsf{RLD}(\operatorname{form} a)}\operatorname{StarComp}(A; F)}{A\in\operatorname{GR} a, F\in\prod_{i\in n}\operatorname{GR} f_i}\right\}\sqcup\bigsqcap\left\{\frac{\uparrow^{\mathsf{RLD}(\operatorname{form} b)}\operatorname{StarComp}(B; F)}{B\in\operatorname{GR} b, F\in\prod_{i\in n}\operatorname{GR} f_i}\right\} = \\
&\bigsqcap\left\{\frac{\uparrow^{\mathsf{RLD}(\operatorname{form} a)}\operatorname{StarComp}(A; F)\sqcup\uparrow^{\mathsf{RLD}(\operatorname{form} b)}\operatorname{StarComp}(B; F)}{A\in\operatorname{GR} a, B\in\operatorname{GR} b, F\in\prod_{i\in n}\operatorname{GR} f_i}\right\} = \\
&\bigsqcap\left\{\frac{\uparrow^{\mathsf{RLD}(\operatorname{form} a)}(\operatorname{StarComp}(A; F)\cup\operatorname{StarComp}(B; F))}{A\in\operatorname{GR} a, B\in\operatorname{GR} b, F\in\prod_{i\in n}\operatorname{GR} f_i}\right\} = \\
&\bigsqcap\left\{\frac{\uparrow^{\mathsf{RLD}(\operatorname{form} a)}\operatorname{StarComp}(A\cup B; F)}{A\in\operatorname{GR} a, B\in\operatorname{GR} b, F\in\prod_{i\in n}\operatorname{GR} f_i}\right\} = \\
&\bigsqcap\left\{\frac{\uparrow^{\mathsf{RLD}(\operatorname{form} a)}\operatorname{StarComp}(C; F)}{C\in\operatorname{GR}(a\sqcup b), F\in\prod_{i\in n}\operatorname{GR} f_i}\right\} = \\
&\operatorname{GR}\operatorname{StarComp}(a\sqcup b; f).
\end{aligned}$$

□