17.12. Multireloids

DEFINITION 1361. I will call a *multireloid* of the form $A = A_{i\in n}$, where every each A_i is a set, a pair $(f; A)$ where f is a filter on the set $\prod A$.

DEFINITION 1362. I will denote $\operatorname{Obj}(f; A) = A$ and $\operatorname{GR}(f; A) = f$ for every multireloid $(f; A)$.

I will denote $\mathsf{RLD}(A)$ the set of multireloids of the form A.

The multireloid $\uparrow^{\mathsf{RLD}(A)} F$ for a relation F is defined by the formulas:FiXme: Should instead be defined for anchored relations.

$$\operatorname{Obj} \uparrow^{\mathsf{RLD}(A)} F = A \quad \text{and} \quad \operatorname{GR} \uparrow^{\mathsf{RLD}(A)} F = \uparrow^{\prod A} F.$$

Let a be a multireloid of the form A and $\operatorname{dom} A = n$.

Let every f_i be a reloid with $\operatorname{Src} f_i = A_i$.

The star-composition of a with f is a multireloid of the form $\lambda i \in \operatorname{dom} A : \operatorname{Dst} f_i$ defined by the formulas:

$$\operatorname{arity} \operatorname{StarComp}(a; f) = n;$$
$$\operatorname{GR} \operatorname{StarComp}(a; f) = \bigsqcap \left\{ \frac{\uparrow^{\mathsf{RLD}(A)} \operatorname{GR} \operatorname{StarComp}(A; F)}{A \in \operatorname{GR} a, F \in \prod_{i\in n} \operatorname{GR} f_i} \right\};$$
$$\operatorname{Obj}_m \operatorname{StarComp}(a; f) = \lambda i \in n : \operatorname{Dst} f_i.$$

THEOREM 1363. Multireloids with above defined compositions form a quasi-invertible category with star-morphisms.

PROOF. We need to prove:

1°. $\operatorname{StarComp}(\operatorname{StarComp}(m; f); g) = \operatorname{StarComp}(m; \lambda i \in \operatorname{arity} m : g_i \circ f_i)$;
2°. $\operatorname{StarComp}(m; \lambda i \in \operatorname{arity} m : \operatorname{id}_{\operatorname{Obj}_m i}) = m$;
3°. $b \not\asymp \operatorname{StarComp}(a; f) \Leftrightarrow a \not\asymp \operatorname{StarComp}(b; f^\dagger)$

(the rest is obvious).

Really,

1°. Using properties of generalized filter bases,

$$\begin{aligned}
&\operatorname{StarComp}(\operatorname{StarComp}(a; f); g) = \\
&\bigsqcap \left\{ \frac{\uparrow^{\mathsf{RLD}} \operatorname{StarComp}(B; G)}{B \in \operatorname{GR} \operatorname{StarComp}(a; f), G \in \prod_{i\in n} \operatorname{GR} g_i} \right\} = \\
&\bigsqcap \left\{ \frac{\uparrow^{\mathsf{RLD}} \operatorname{StarComp}(\operatorname{StarComp}(A; F); G)}{A \in \operatorname{GR} a, F \in \prod_{i\in n} \operatorname{GR} f_i, G \in \prod_{i\in n} \operatorname{GR} g_i} \right\} = \\
&\bigsqcap \left\{ \frac{\uparrow^{\mathsf{RLD}} \operatorname{StarComp}(A; G \circ F)}{A \in \operatorname{GR} a, F \in \prod_{i\in n} \operatorname{GR} f_i, G \in \prod_{i\in n} \operatorname{GR} g_i} \right\} = \\
&\bigsqcap \left\{ \frac{\uparrow^{\mathsf{RLD}} \operatorname{StarComp}(A; H)}{A \in \operatorname{GR} a, H \in \prod_{i\in n} \lambda i \in n : g_i \circ f_i} \right\} = \\
&\operatorname{StarComp}(a; \lambda i \in \operatorname{arity} n : g_i \circ f_i).
\end{aligned}$$