PROPOSITION 1354. Simple product is a pointfree funcoid

$$\prod^{(S)} f \in \mathsf{FCD}\left(\prod_{i\in\operatorname{dom} f} \operatorname{Src} f_i; \prod_{i\in\operatorname{dom} f} \operatorname{Dst} f_i\right).$$

PROOF. Let $x \in \prod_{i\in\operatorname{dom} f} \operatorname{Src} f_i$ and $y \in \prod_{i\in\operatorname{dom} f} \operatorname{Dst} f_i$. Then (take into account that $\operatorname{Src} f_i$ and $\operatorname{Dst} f_i$ are posets with least elements)

$$y \not\asymp \left(\lambda x \in \prod_{i\in\operatorname{dom} f} \operatorname{Src} f_i : \lambda i \in \operatorname{dom} f : \langle f_i\rangle x_i\right) x \Leftrightarrow$$
$$y \not\asymp \lambda i \in \operatorname{dom} f : \langle f_i\rangle x_i \Leftrightarrow$$
$$\exists i \in \operatorname{dom} f : y_i \not\asymp \langle f_i\rangle x_i \Leftrightarrow$$
$$\exists i \in \operatorname{dom} f : x_i \not\asymp \left\langle f_i^{-1}\right\rangle y_i \Leftrightarrow$$
$$x \not\asymp \lambda i \in \operatorname{dom} f : \left\langle f_i^{-1}\right\rangle y_i \Leftrightarrow$$
$$x \not\asymp \left(\lambda y \in \prod_{i\in\operatorname{dom} f} \operatorname{Dst} f_i : \lambda i \in \operatorname{dom} f : \left\langle f_i^{-1}\right\rangle y_i\right) y.$$

□

OBVIOUS 1355. $\left\langle \prod^{(S)} f\right\rangle x = \lambda i \in \operatorname{dom} f : \langle f_i\rangle x_i$ for $x \in \prod \operatorname{Src} f_i$.

OBVIOUS 1356. $\left(\left\langle \prod^{(S)} f\right\rangle x\right)_i = \langle f_i\rangle x_i$ for $x \in \prod \operatorname{Src} f_i$.

PROPOSITION 1357. f_i can be restored if we know $\prod^{(S)} f$ if f_i is a family of pointfree funcoids between posets with least elements.

PROOF. Let's restore the value of $\langle f_i\rangle x$ where $i \in \operatorname{dom} f$ and $x \in \operatorname{Src} f_i$.
Let $x'_i = x$ and $x'_j = 0$ for $j \neq i$.
Then $\langle f_i\rangle x = \langle f_i\rangle x'_i = \left(\left\langle \prod^{(S)} f\right\rangle x'\right)_i$.
We have restored the value of $\langle f_i\rangle$. Restoring the value of $\left\langle f_i^{-1}\right\rangle$ is similar. □

REMARK 1358. In the above proposition it is not required that f_i are non-zero.

PROPOSITION 1359. $\left(\prod^{(S)} g\right) \circ \left(\prod^{(S)} f\right) = \prod^{(S)}_{i\in n}(g_i \circ f_i)$ for n-indexed families f and g of composable pointfree funcoids between posets with least elements.

PROOF.

$$\left\langle \prod^{(S)}_{i\in n}(g_i \circ f_i)\right\rangle x = \lambda i \in \operatorname{dom} f : \langle g_i \circ f_i\rangle x_i = \lambda i \in \operatorname{dom} f : \langle g_i\rangle\langle f_i\rangle x_i =$$
$$\left\langle \prod^{(S)} g\right\rangle \lambda i \in \operatorname{dom} f : \langle f_i\rangle x_i = \left\langle \prod^{(S)} g\right\rangle\left\langle \prod^{(S)} f\right\rangle x = \left\langle \left(\prod^{(S)} g\right) \circ \left(\prod^{(S)} f\right)\right\rangle x.$$

Thus $\left\langle \prod^{(S)}_{i\in n}(g_i \circ f_i)\right\rangle = \left\langle \left(\prod^{(S)} g\right) \circ \left(\prod^{(S)} f\right)\right\rangle$.
$\left\langle \left(\prod^{(S)}_{i\in n}(g_i \circ f_i)\right)^{-1}\right\rangle = \left\langle \left(\left(\prod^{(S)} g\right) \circ \left(\prod^{(S)} f\right)\right)^{-1}\right\rangle$ is similar. □

COROLLARY 1360. $\left(\prod^{(S)} f_{k-1}\right) \circ \ldots \circ \left(\prod^{(S)} f_0\right) = \prod^{(S)}_{i\in n}(f_{k-1} \circ \ldots \circ f_0)$ for every n-indexed families $f_0, \ldots, f_{n-1}$ of composable pointfree funcoids between posets with least elements.