2°. Obvious.
3°. It follows from the proposition above.

□

OBVIOUS 1340. $\operatorname{StarComp}(a \cup b; f) = \operatorname{StarComp}(a; f) \cup \operatorname{StarComp}(b; f)$ for n-ary relations a, b and an n-indexed family f of binary relations.

THEOREM 1341. $\left\langle \prod^{(C)} f \right\rangle \prod a = \prod_{i \in n} \langle f_i \rangle^* a_i$ for every family $f = f_{i \in n}$ of binary relations and $a = a_{i \in n}$ where a_i is a small set (for each $i \in n$).

PROOF.

$$L \in \left\langle \prod^{(C)} f \right\rangle \prod a \Leftrightarrow$$
$$L \in \operatorname{StarComp}\left(\prod a; f\right) \Leftrightarrow$$
$$\exists y \in \prod a \forall i \in n : y_i \; f_i \; L_i \Leftrightarrow$$
$$\exists y \in \prod a \forall i \in n : \{y_i\} \not\asymp \left\langle f_i^{-1} \right\rangle^* \{L_i\} \Leftrightarrow$$
$$\forall i \in n \exists y \in a_i : \{y\} \not\asymp \left\langle f_i^{-1} \right\rangle^* \{L_i\} \Leftrightarrow$$
$$\forall i \in n : a_i \not\asymp \left\langle f_i^{-1} \right\rangle^* \{L_i\} \Leftrightarrow$$
$$\forall i \in n : \{L_i\} \not\asymp \langle f_i \rangle^* a_i \Leftrightarrow$$
$$\forall i \in n : L_i \in \langle f_i \rangle^* a_i \Leftrightarrow$$
$$L \in \prod_{i \in n} \langle f_i \rangle^* a_i.$$

□

17.11.4. Star composition of Rel-morphisms. Define *star composition* for an n-ary anchored relation a and an n-indexed family f of **Rel**-morphisms as an n-ary anchored relation complying with the formulas:

$$\operatorname{Obj}_{\operatorname{StarComp}(a;f)} = \lambda i \in \operatorname{arity} a : \operatorname{Dst} f_i;$$
$$\operatorname{arity} \operatorname{StarComp}(a; f) = \operatorname{arity} a;$$
$$L \in \operatorname{GR} \operatorname{StarComp}(a; f) \Leftrightarrow L \in \operatorname{StarComp}(\operatorname{GR} a; \operatorname{GR} \circ f).$$

(Here I denote $GR(A; B; f) = f$ for every **Rel**-morphism f.)

PROPOSITION 1342. $b \not\asymp \operatorname{StarComp}(a; f) \Leftrightarrow \exists x \in a, y \in b \forall j \in n : x_j \; f_j \; y_j$.

PROOF. From the previous section. □

THEOREM 1343. Relations with above defined compositions form a quasi-invertible category with star-morphisms.

PROOF. We need to prove:

1°. $\operatorname{StarComp}(\operatorname{StarComp}(m; f); g) = \operatorname{StarComp}(m; \lambda i \in \operatorname{arity} m : g_i \circ f_i)$;
2°. $\operatorname{StarComp}(m; \lambda i \in \operatorname{arity} m : \operatorname{id}_{\operatorname{Obj}_m i}) = m$;
3°. $b \not\asymp \operatorname{StarComp}(a; f) \Leftrightarrow a \not\asymp \operatorname{StarComp}(b; f^\dagger)$

(the rest is obvious).

It follows from the previous section. □

PROPOSITION 1344. $\operatorname{StarComp}(a \sqcup b; f) = \operatorname{StarComp}(a; f) \sqcup \operatorname{StarComp}(b; f)$ for an n-ary anchored relations a, b and an n-indexed family f of **Rel**-morphisms.

PROOF. It follows from the previous section. □