- Compositions of morphisms are defined by the formulas:
  - $g \circ f = \lambda i \in \operatorname{dom} f : g_i \circ f_i$ for our indexed families f and g of morphisms;
  - $f \circ m = \operatorname{StarComp}(m; f)$ for $m \in M$ and a composable indexed family f;
  - $m \circ \mathrm{id}_{\mathrm{None}} = m$ for $m \in M$;
  - $\mathrm{id}_{\mathrm{None}} \circ \mathrm{id}_{\mathrm{None}} = \mathrm{id}_{\mathrm{None}}$.
- Identity morphisms for an object X are:
  - $\lambda i \in X : \mathrm{id}_{X_i}$ if $X \neq \mathrm{None}$;
  - $\mathrm{id}_{\mathrm{None}}$ if $X = \mathrm{None}$.

Proof. We need to prove it is really a category.

We need to prove:

1°. Composition is associative.

2°. Composition with identities complies with the identity law.

Really:

1°. $(h \circ g) \circ f = \lambda i \in \operatorname{dom} f : (h_i \circ g_i) \circ f_i = \lambda i \in \operatorname{dom} f : h_i \circ (g_i \circ f_i) = h \circ (g \circ f)$;

$$g \circ (f \circ m) = \operatorname{StarComp}(\operatorname{StarComp}(m; f); g) = \\ \operatorname{StarComp}(m; \lambda i \in \operatorname{arity} m : g_i \circ f_i) = \operatorname{StarComp}(m; g \circ f) = (g \circ f) \circ m;$$

$f \circ (m \circ \mathrm{id}_{\mathrm{None}}) = f \circ m = (f \circ m) \circ \mathrm{id}_{\mathrm{None}}$.

2°. $m \circ \mathrm{id}_{\mathrm{None}} = m$; $\mathrm{id}_{\operatorname{Dst} m} \circ m = \operatorname{StarComp}(m; \lambda i \in \operatorname{arity} m : \mathrm{id}_{\mathrm{Obj}_m i}) = m$.

□

Remark 1327. I call the above defined category *abrupt category* because (excluding identity morphisms) it allows composition with an $m \in M$ only on the left (not on the right) so that the morphism m is "abrupt" on the right.

FiXme: Need to be defined easier. Is it defined earlier?By $[\![x_0; \ldots; x_{n-1}]\!]$ I denote an n-tuple.

Definition 1328. Precategory with star morphisms *induced* by a dagger precategory C is:

- The base category is C.
- Star-morphisms are morphisms of C.
- $\operatorname{arity} f = \{0, 1\}$.
- $\mathrm{Obj}_m = [\![\operatorname{Src} m; \operatorname{Dst} m]\!]$.
- $\operatorname{StarComp}(m; [\![f; g]\!]) = g \circ m \circ f^\dagger$.

Let prove it is really a precategory with star-morphisms.

Proof. We need to prove the associativity law:

$$\operatorname{StarComp}(\operatorname{StarComp}(m; [\![f; g]\!]); [\![p; q]\!]) = \operatorname{StarComp}(m; [\![p \circ f; q \circ g]\!]).$$

Really,

$$\operatorname{StarComp}(\operatorname{StarComp}(m; [\![f; g]\!]); [\![p; q]\!]) = \operatorname{StarComp}(g \circ m \circ f^\dagger; [\![p; q]\!]) = \\ q \circ g \circ m \circ f^\dagger \circ p^\dagger = q \circ g \circ m \circ (p \circ f)^\dagger = \operatorname{StarComp}(m; [\![p \circ f; q \circ g]\!]).$$

□

Definition 1329. Category with star morphisms *induced* by a dagger category C is the above defined precategory with star-morphisms.

That it is a category (the law of composition with identity) is trivial.