PROOF. Use the fact that $\mathrm{GR}\prod^{(\mathrm{ord})} F = \left\{ \frac{F\circ(\bigoplus(\mathrm{dom}\circ F))^{-1}}{F\in\mathrm{GR}\prod^{(D)} f} \right\}$. □

DEFINITION 1316. $f \times^{(\mathrm{ord})} g = \prod^{(\mathrm{ord})}[\![f; g]\!]$.

REMARK 1317. If f and g are binary funcoids, then $f \times^{(\mathrm{ord})} g$ is ternary.

PROPOSITION 1318. FiXme: Duplicate with 1296$\prod^{\mathsf{Strd}} a = \upuparrows\downdownarrows \prod^{\mathsf{Strd}} a$ if each $a_i \in \mathfrak{A}_i$ (for $i \in n$ where n is some index set) where each $(\mathfrak{A}_{i\in n}; \mathfrak{Z}_{i\in n})$ is a down aligned filtrator with separable core.

PROOF.

$$\mathrm{GR} \upuparrows\downdownarrows \prod^{\mathsf{Strd}} a =$$
$$\left\{ \frac{L \in \prod \mathfrak{A}}{\mathrm{up}\, L \subseteq \mathfrak{Z} \cap \mathrm{GR} \prod^{\mathsf{Strd}} a} \right\} =$$
$$\left\{ \frac{L \in \prod \mathfrak{A}}{\mathrm{up}\, L \subseteq \mathrm{GR} \prod^{\mathsf{Strd}} a} \right\} =$$
$$\left\{ \frac{L \in \prod \mathfrak{A}}{\forall K \in \mathrm{up}\, L : K \in \mathrm{GR} \prod^{\mathsf{Strd}} a} \right\} \Leftrightarrow$$
$$\left\{ \frac{L \in \prod \mathfrak{A}}{\forall K \in \mathrm{up}\, L, i \in n : K_i \not\asymp a_i} \right\} \Leftrightarrow$$
$$\left\{ \frac{L \in \prod \mathfrak{A}}{\forall i \in n, K \in \mathrm{up}\, L : K_i \not\asymp a_i} \right\} =$$
$$\left\{ \frac{L \in \prod \mathfrak{A}}{\forall i \in n : L_i \not\asymp a_i} \right\} =$$
$$\mathrm{GR} \prod^{\mathsf{Strd}} a$$

(taken into account that our filtrators is with a separable core). □

17.10. Star categories

DEFINITION 1319. A *precategory with star-morphisms* consists of

1°. a precategory C (*the base precategory*);
2°. a set M (*star-morphisms*);
3°. a function "arity" defined on M (how many objects are connected by this star-morphism);
4°. a function $\mathrm{Obj}_m : \mathrm{arity}\, m \to \mathrm{Obj}(C)$ defined for every $m \in M$;
5°. a function (*star composition*) $(m; f) \mapsto \mathrm{StarComp}(m; f)$ defined for $m \in M$ and f being an $(\mathrm{arity}\, m)$-indexed family of morphisms of C such that $\forall i \in \mathrm{arity}\, m : \mathrm{Src}\, f_i = \mathrm{Obj}_m\, i$ ($\mathrm{Src}\, f_i$ is the source object of the morphism f_i) such that $\mathrm{arity}\,\mathrm{StarComp}(m; f) = \mathrm{arity}\, m$

such that it holds:

1°. $\mathrm{StarComp}(m; f) \in M$;
2°. (*associativity law*)

$$\mathrm{StarComp}(\mathrm{StarComp}(m; f); g) = \mathrm{StarComp}(m; \lambda i \in \mathrm{arity}\, m : g_i \circ f_i).$$

The meaning of the set M is an extension of C having as morphisms things with arbitrary (possibly infinite) indexed set Obj_m of objects, not just two objects as morphisms of C have only source and destination.