1°. $\operatorname{form} \prod^{(D)} F = \left\{ \frac{((i;j);(\operatorname{form} F_i)_j)}{i \in \operatorname{dom} F, j \in \operatorname{arity} F_i} \right\}$;
2°. $\operatorname{GR} \prod^{(D)} F = \left\{ \frac{\left\{ \frac{((i;j);(zi)j)}{i \in \operatorname{dom} F, j \in \operatorname{arity} F_i} \right\}}{z \in \prod (\operatorname{GR} \circ F)} \right\}$.

PROPOSITION 1309. $\prod^{(D)} F$ is an anchored relation if every F_i is an anchored relation.

PROOF. We need to prove $\operatorname{GR} \prod^{(D)} F \in \mathscr{P} \prod \operatorname{form}\left(\prod^{(D)} F\right)$ that is $\operatorname{GR} \prod^{(D)} F \subseteq \prod \operatorname{form}\left(\prod^{(D)} F\right)$; $\left\{ \frac{\left\{ \frac{((i;j);(zi)j)}{i \in \operatorname{dom} F, j \in \operatorname{arity} F_i} \right\}}{z \in \prod (\operatorname{GR} \circ F)} \right\} \subseteq \prod \left\{ \frac{((i;j);(\operatorname{form} F_i)_j)}{i \in \operatorname{dom} F, j \in \operatorname{arity} F_i} \right\}$;
$\forall z \in \prod (\operatorname{GR} \circ F), i \in \operatorname{dom} F, j \in \operatorname{arity} F_i : (zi)j \in (\operatorname{form} F_i)_j$.
Really, $zi \in \operatorname{GR} F_i \subseteq \prod (\operatorname{form} F_i)$ and thus $(zi)j \in (\operatorname{form} F_i)_j$. □

OBVIOUS 1310. $\operatorname{arity} \prod^{(D)} F = \coprod_{i \in \operatorname{dom} F} \operatorname{arity} F_i = \left\{ \frac{(i;j)}{i \in \operatorname{dom} F, j \in \operatorname{arity} F_i} \right\}$.

DEFINITION 1311. $f \times^{(D)} g = \prod^{(D)} [\![f; g]\!]$.

LEMMA 1312. $\prod^{(D)} F$ is an upper set if every F_i is an upper set.

PROOF. We need to prove that $\prod^{(D)} F$ is an upper set. Let $a \in \prod^{(D)} F$ and an anchored relation $b \sqsupseteq a$ of the same form as a. We have $a = \operatorname{uncurry} z$ for some $z \in \prod F$ that is $a(i; j) = (zi)j$ for all $i \in \operatorname{dom} F$ and $j \in \operatorname{dom} F_i$ where $zi \in F_i$. Also $b(i; j) \sqsupseteq a(i; j)$. Thus $(\operatorname{curry} b)i \sqsupseteq zi$; $\operatorname{curry} b \in \prod F$ because every F_i is an upper set and so $b \in \prod^{(D)} F$. □

PROPOSITION 1313. Let F be an indexed family of anchored relations and every $(\operatorname{form} F)_i$ is a join-semilattice.

1°. $\prod^{(D)} F$ is a prestaroid if every F_i is a prestaroid.
2°. $\prod^{(D)} F$ is a staroid if every F_i is a staroid.
3°. $\prod^{(D)} F$ is a completary staroid if every F_i is a completary staroid.

PROOF.

1°. Let $q \in \operatorname{arity} \prod^{(D)} F$ that is $q = (i; j)$ where $i \in \operatorname{dom} F$, $j \in \operatorname{arity} F_i$; let

$$L \in \prod \left(\left(\operatorname{form} \prod^{(D)} F \right) |_{\left(\operatorname{arity} \prod^{(D)} F\right) \setminus \{q\}} \right)$$