Proof.

$$\begin{aligned}
\operatorname{GR} \downarrow\downarrow \prod^{\mathsf{Strd}(\mathfrak{A})} A &= \\
\downarrow\downarrow \operatorname{GR} \prod^{\mathsf{Strd}(\mathfrak{A})} A &= \\
\downarrow\downarrow \left\{ \frac{L \in \prod \mathfrak{A}}{\forall i \in \operatorname{dom} \mathfrak{A} : A_i \not\asymp L_i} \right\} &= \\
\left\{ \frac{L \in \prod \mathfrak{A}}{\forall i \in \operatorname{dom} \mathfrak{A} : A_i \not\asymp L_i} \right\} \cap \mathfrak{Z} &= \\
\left\{ \frac{L \in \prod \mathfrak{Z}}{\forall i \in \operatorname{dom} \mathfrak{A} : A_i \not\asymp L_i} \right\} &= \\
\operatorname{GR} \prod^{\mathsf{Strd}(\mathfrak{Z})} A.
\end{aligned}$$

□

Corollary 1303. If each $(\mathfrak{F}_i; \mathfrak{P}_i)$ is a powerset filtrator and $A \in \prod \mathfrak{P}$, then $\downarrow\downarrow \prod^{\mathsf{Strd}(\mathfrak{F})} A$ is a principal staroid.

Proof. Use the "obvious" fact above. □

Theorem 1304. Let $\mathfrak{F}$ be a family of sets of filters on distributive lattices with least elements. Let $a \in \prod \mathfrak{F}$, $S \in \mathscr{P} \prod \mathfrak{F}$, and every $\operatorname{Pr}_i S$ be a generalized filter base, $\bigsqcap S = a$. Then

$$\prod^{\mathsf{Strd}(\mathfrak{F})} a = \bigsqcap \left\{ \frac{\prod^{\mathsf{Strd}(\mathfrak{F})} A}{A \in S} \right\}.$$

Proof. That $\prod^{\mathsf{Strd}(\mathfrak{F})} a$ is a lower bound for $\left\{ \frac{\prod^{\mathsf{Strd}(\mathfrak{F})} A}{A \in S} \right\}$ is obvious.

Let f be a lower bound for $\left\{ \frac{\prod^{\mathsf{Strd}(\mathfrak{F})} A}{A \in S} \right\}$. Thus $\forall A \in S : \operatorname{GR} f \subseteq \operatorname{GR} \prod^{\mathsf{Strd}(\mathfrak{F})} A$. Thus for every $A \in S$ we have $L \in \operatorname{GR} f$ implies $\forall i \in \operatorname{dom} \mathfrak{A} : A_i \not\asymp L_i$. Then, by properties of generalized filter bases, $\forall i \in \operatorname{dom} \mathfrak{A} : a_i \not\asymp L_i$ that is $L \in \operatorname{GR} \prod^{\mathsf{Strd}(\mathfrak{F})} a$.

So $f \sqsubseteq \prod^{\mathsf{Strd}(\mathfrak{F})} a$. □

Conjecture 1305. Let $\mathfrak{F}$ be a family of sets of filters on distributive lattices with least elements. Let $a \in \prod \mathfrak{F}$, $S \in \mathscr{P} \prod \mathfrak{F}$ be a generalized filter base, $\bigsqcap S = a$, f is a staroid of the form $\prod \mathfrak{F}$. Then

$$\prod^{\mathsf{Strd}(\mathfrak{F})} a \not\asymp f \Leftrightarrow \forall A \in S : \prod^{\mathsf{Strd}(\mathfrak{A})} A \not\asymp f.$$

17.9. On products of staroids

Definition 1306. $\prod^{(D)} F = \left\{ \frac{\operatorname{uncurry} z}{z \in \prod F} \right\}$ (*reindexation product*) for every indexed family F of relations.

Definition 1307. *Reindexation product* of an indexed family F of anchored relations is defined by the formulas:

$$\operatorname{form} \prod^{(D)} F = \operatorname{uncurry}(\operatorname{form} \circ F) \quad \text{and} \quad \operatorname{GR} \prod^{(D)} F = \prod^{(D)} (\operatorname{GR} \circ F).$$

Obvious 1308.