THEOREM 1288. $\bigsqcup^{\mathsf{pFCD}(\mathfrak{A})} F = \bigsqcup F$ for every set F of premultifuncoids for the same indexed family of join infinite distributive complete lattices filtrators.

PROOF. $\alpha_i x \overset{\text{def}}{=} \bigsqcup_{f \in F} f_i x$. It is enough to prove that α is a premultifuncoid. We need to prove:

$$L_i \not\asymp \alpha_i L|_{(\operatorname{dom} L)\setminus\{i\}} \Leftrightarrow L_j \not\asymp \alpha_j L|_{(\operatorname{dom} L)\setminus\{j\}}.$$

Really,

$$\begin{aligned}
L_i \not\asymp \alpha_i L|_{(\operatorname{dom} L)\setminus\{i\}} &\Leftrightarrow \\
L_i \not\asymp \bigsqcup_{f \in F} f_i L|_{(\operatorname{dom} L)\setminus\{i\}} &\Leftrightarrow \\
\exists f \in F : L_i \not\asymp f_i L|_{(\operatorname{dom} L)\setminus\{i\}} &\Leftrightarrow \\
\exists f \in F : L_j \not\asymp f_j L|_{(\operatorname{dom} L)\setminus\{j\}} &\Leftrightarrow \\
L_j \not\asymp \bigsqcup_{f \in F} f_j L|_{(\operatorname{dom} L)\setminus\{j\}} &\Leftrightarrow \\
L_j \not\asymp \alpha_j L|_{(\operatorname{dom} L)\setminus\{j\}}.&
\end{aligned}$$

□

PROPOSITION 1289. The mapping $f \mapsto [f]$ is an order embedding, for multifuncoids for indexed families $(\mathfrak{A}_i; \mathfrak{Z}_i)$ of down-aligned starrish filtrators with separable finitely meet-closed core.

PROOF. The mapping $f \mapsto [f]$ is defined because $\mathfrak{A}_i$ are starrish posets (and $(\mathfrak{A}_i; \mathfrak{Z}_i)$ is with finitely meet-closed core and down-aligned). The mapping is injective because the filtrators are with separable cores ($\left\{ \frac{X \in \mathfrak{Z}_i}{X \not\asymp \langle f \rangle A} \right\} = \left\{ \frac{X \in \mathfrak{Z}_i}{X \not\asymp \langle f \rangle B} \right\}$ implies $\langle f \rangle A = \langle f \rangle B$). That $f \mapsto [f]$ is a monotone function is obvious. □

REMARK 1290. This order embedding is useful to describe properties of posets of prestaroids.

THEOREM 1291. If f, g are multifuncoids for the filtrator $(\mathfrak{F}_i; \mathfrak{P}_i)$ where $\mathfrak{Z}_i$ are separable starrish posets, then $f \sqcup^{\mathsf{pFCD}(\mathfrak{A})} g \in \mathsf{FCD}(\mathfrak{A})$.

PROOF. Let $A \in \left[f \sqcup^{\mathsf{pFCD}(\mathfrak{A})} g\right]$ and $B \sqsupseteq A$. Then for every $k \in \operatorname{dom} \mathfrak{A}$

$A_k \not\asymp (f \sqcup^{\mathsf{pFCD}(\mathfrak{A})} g)A|_{(\operatorname{dom} \mathfrak{A})\setminus\{k\}}$; $A_k \not\asymp (f \sqcup g)A|_{(\operatorname{dom} \mathfrak{A})\setminus\{k\}}$; $A_k \not\asymp f(A|_{(\operatorname{dom} \mathfrak{A})\setminus\{k\}}) \sqcup g(A|_{(\operatorname{dom} \mathfrak{A})\setminus\{k\}})$.

Thus $A_k \not\asymp f(A|_{(\operatorname{dom} \mathfrak{A})\setminus\{k\}}) \vee A_k \not\asymp g(A|_{(\operatorname{dom} \mathfrak{A})\setminus\{k\}})$; $A \in [f] \vee A \in [g]$; $B \in [f] \vee B \in [g]$; $B_k \not\asymp f(B|_{(\operatorname{dom} \mathfrak{A})\setminus\{k\}}) \vee B_k \not\asymp g(B|_{(\operatorname{dom} \mathfrak{A})\setminus\{k\}})$;

$B_k \not\asymp f(B|_{(\operatorname{dom} \mathfrak{A})\setminus\{k\}}) \sqcup g(B|_{(\operatorname{dom} \mathfrak{A})\setminus\{k\}})$; $B_k \not\asymp (f \sqcup g)B|_{(\operatorname{dom} \mathfrak{A})\setminus\{k\}} = (f \sqcup^{\mathsf{pFCD}(\mathfrak{A})} g)B|_{(\operatorname{dom} \mathfrak{A})\setminus\{k\}}$.

Thus $B \in \left[f \sqcup^{\mathsf{pFCD}(\mathfrak{A})} g\right]$. □

THEOREM 1292. If F is a set of multifuncoids for the same indexed family of join infinite distributive complete lattices filtrators, then $\bigsqcup^{\mathsf{pFCD}(\mathfrak{A})} F \in \mathsf{FCD}(\mathfrak{A})$.

PROOF. Let $A \in \left[\bigsqcup^{\mathsf{pFCD}(\mathfrak{A})} F\right]$ and $B \sqsupseteq A$. Then for every $k \in \operatorname{dom} \mathfrak{A}$

$A_k \not\asymp \left(\bigsqcup^{\mathsf{pFCD}(\mathfrak{A})} F\right) A|_{(\operatorname{dom} \mathfrak{A})\setminus\{k\}} = (\bigsqcup F)A|_{(\operatorname{dom} \mathfrak{A})\setminus\{k\}} = \bigsqcup_{f \in F} f(A|_{(\operatorname{dom} \mathfrak{A})\setminus\{k\}})$.

Thus $\exists f \in F : A_k \not\asymp f(A|_{(\operatorname{dom} \mathfrak{A})\setminus\{k\}})$; $\exists f \in F : A \in [f]$; $B \in [f]$ for some $f \in F$; $\exists f \in F : B_k \not\asymp f(B|_{(\operatorname{dom} \mathfrak{A})\setminus\{k\}})$; $B_k \not\asymp \bigsqcup_{f \in F} f(B|_{(\operatorname{dom} \mathfrak{A})\setminus\{k\}}) = \left(\bigsqcup^{\mathsf{pFCD}(\mathfrak{A})} F\right) B|_{(\operatorname{dom} \mathfrak{A})\setminus\{k\}}$. Thus $B \in \left[\bigsqcup^{\mathsf{pFCD}(\mathfrak{A})} F\right]$. □