THEOREM 1279. Fix some indexed family $\mathfrak{A}$ of boolean lattices. The the set of premultifuncoids g for the filtrator $(\mathfrak{F}_i; \mathfrak{P}_i)$ bijectively corresponds to set of prestaroids f of form $\mathfrak{P} = \lambda i \in \operatorname{dom} \mathfrak{A} : \mathfrak{P}_i$ by the formulas:

1°. $f = [g]$;
2°. $\partial \langle g \rangle_i L = (\operatorname{val} f)_i L$ for every $i \in \operatorname{dom} \mathfrak{A}$, $L \in \prod \mathfrak{P}|_{\operatorname{dom} \mathfrak{A} \setminus \{i\}}$.

PROOF. Let f be a prestaroid of the form $\mathfrak{P}$. If α is defined by the formula $\alpha_i L = \langle f \rangle_i L$ then $\partial \alpha_i L = (\operatorname{val} f)_i L$. Then

$$L_i \not\asymp \alpha_i L|_{(\operatorname{dom} L) \setminus \{i\}} \Leftrightarrow L \in f \Leftrightarrow L_j \not\asymp \alpha_j L|_{(\operatorname{dom} L) \setminus \{j\}}.$$

For the prestaroid f' defined by the formula $L \in f' \Leftrightarrow L_i \not\asymp \alpha_i L|_{(\operatorname{dom} L) \setminus \{i\}}$ we have:

$$L \in f' \Leftrightarrow L_i \in \partial \alpha_i L|_{(\operatorname{dom} L) \setminus \{i\}} \Leftrightarrow L_i \in (\operatorname{val} f)_i L|_{(\operatorname{dom} L) \setminus \{i\}} \Leftrightarrow L \in f;$$

thus $f' = f$.

Let now α be an indexed family of functions $\alpha_i \in \mathfrak{F}(\mathfrak{Z}_i)^{(\operatorname{dom} \mathfrak{Z}) \setminus \{i\}}$ conforming to the formula (26). Let relation f between posets be defined by the formula $L \in f \Leftrightarrow L_i \not\asymp \alpha_i L|_{(\operatorname{dom} L) \setminus \{i\}}$. Then

$$(\operatorname{val} f)_i L = \left\{ \frac{K \in \mathfrak{P}_i}{K \not\asymp \alpha_i L|_{(\operatorname{dom} L) \setminus \{i\}}} \right\} = \partial \alpha_i L|_{(\operatorname{dom} L) \setminus \{i\}}$$

and thus $(\operatorname{val} f)_i L$ is a core star that is f is a prestaroid. For the indexed family α' defined by the formula $\alpha'_i L = \langle f \rangle_i L$ we have

$$\partial \alpha'_i L = \partial \langle f \rangle_i L = \left\{ \frac{K \in \mathfrak{P}_i}{K \not\asymp \alpha_i L} \right\} = \partial \alpha_i L;$$

thus $\alpha' = \alpha$ (taking into account that $\mathfrak{P}_i$ is a boolean lattice).

We have shown that these are bijections. □

DEFINITION 1280. I will denote Λf the premultifuncoid corresponding to a prestaroid f (for an indexed family of boolean lattices) by the above theorem.

THEOREM 1281. Fix some indexed family $\mathfrak{Z}$ of boolean lattices. $\langle f \rangle_j (L \cup \{(i; X \sqcup Y)\}) = \langle f \rangle_j (L \cup \{(i; X)\}) \sqcup \langle f \rangle_j (L \cup \{(i; Y)\})$ for every premultifuncoid f for the family $(\mathfrak{F}_i; \mathfrak{P}_i)$ of filtrators and $i, j \in \operatorname{arity} f$, $i \neq j$, $L \in \prod_{k \in L \setminus \{i,j\}} \mathfrak{Z}_k$, $X, Y \in \mathfrak{A}_i$.FiXme: It also holds for any finite number of arguments.

PROOF. Let $i \in \operatorname{arity} f$ and $L \in \prod_{k \in L \setminus \{i,j\}} \mathfrak{Z}_k$. Let $Z \in \mathfrak{Z}_i$.

$$\begin{aligned} Z \not\asymp \langle f \rangle_j (L \cup \{(i; X \sqcup Y)\}) \Leftrightarrow \\ L \cup \{(i; X \sqcup Y), (j; Z)\} \in f \Leftrightarrow \\ X \sqcup Y \in (\operatorname{val} f)_i (L \cup \{(j; Z)\}) \Leftrightarrow \\ X \in (\operatorname{val} f)_i (L \cup \{(j; Z)\}) \vee Y \in (\operatorname{val} f)_i (L \cup \{(j; Z)\}) \Leftrightarrow \\ L \cup \{(i; X), (j; Z)\} \in [f] \vee L \cup \{(i; Y), (j; Z)\} \in [f] \Leftrightarrow \\ Z \not\asymp \langle f \rangle_j (L \cup \{(i; X)\}) \vee Z \not\asymp \langle f \rangle_j (L \cup \{(i; Y)\}). \end{aligned}$$

Thus $\langle f \rangle_j (L \cup \{(i; X \sqcup Y)\}) = \langle f \rangle_j (L \cup \{(i; X)\}) \sqcup \langle f \rangle_j (L \cup \{(i; Y)\})$. □

Let us consider the filtrator $\left(\prod_{i \in \operatorname{arity} f} \mathfrak{F}((\operatorname{form} f)_i); \prod_{i \in \operatorname{arity} f} (\operatorname{form} f)_i \right)$.

THEOREM 1282. Let $(\mathfrak{A}_i; \mathfrak{Z}_i)$ be a family of join-closed down-aligned filtrators whose both base and core are join-semilattices. Let f be a staroid of the form $\mathfrak{Z}$. Then $\upuparrows f$ is a staroid of the form $\mathfrak{A}$.