

Really

$$\begin{aligned}
& \prod (L_0 \sqcup L_1) \not\asymp F \Leftrightarrow \\
& \exists x \in \prod (L_0 \sqcup L_1) : x \in F \Leftrightarrow \\
& \exists x \in \prod_{i \in \text{arity } f} (\text{form } f)_i \forall i \in \text{arity } f : (x_i \in L_0 i \cup L_1 i \wedge x \in F) \Leftrightarrow \\
& \exists x \in \prod_{i \in \text{arity } f} (\text{form } f)_i \forall i \in \text{arity } f : ((x_i \in L_0 i \vee x_i \in L_1 i) \wedge x \in F) \Leftrightarrow \\
& \exists x \in \prod_{i \in \text{arity } f} (\text{form } f)_i \left(\exists c \in \{0, 1\}^{\text{arity } f} : x \in \prod_{i \in \text{arity } f} L_{c(i)} i \wedge x \in F \right) \Leftrightarrow \\
& \exists c \in \{0, 1\}^{\text{arity } f} : \prod_{i \in n} L_{c(i)} i \not\asymp F.
\end{aligned}$$

□

DEFINITION 1268. The *upgraded staroid generated* by an anchored relation F is the staroid $\uparrow\uparrow^{\text{Strd}} F$.

PROPOSITION 1269. $\uparrow^{\text{Strd}} F = \downarrow\downarrow \uparrow\uparrow^{\text{Strd}} F$.

PROOF. Because $\text{GR } \uparrow^{\text{Strd}} F$ is an upper set. □

CONJECTURE 1270. Every upgraded principal staroid is a completary staroid.

CONJECTURE 1271. Filtrators of staroids on powersets are join-closed.

17.6. Multifuncoids

DEFINITION 1272. Let $(\mathfrak{A}_i; \mathfrak{Z}_i)$ (where $i \in n$ for an index set n) be an indexed family of filtrators.

I call a *premultifuncoid sketch* f of the form $(\mathfrak{A}_i; \mathfrak{Z}_i)$ the n -indexed family α of functions where for every $i \in n$

$$\alpha_i : \prod \mathfrak{Z}|_{(\text{dom } \mathfrak{A}) \setminus \{i\}} \rightarrow \mathfrak{A}_i.$$

I denote $\langle f \rangle = \alpha$. **FixMe: Should be $\langle f \rangle^*$.**

DEFINITION 1273. A *premultifuncoid sketch on powersets* is a premultifuncoid sketch such that every $(\mathfrak{A}_i; \mathfrak{Z}_i)$ is the primary filtrator of filters on a powerset.

DEFINITION 1274. I will call a *premultifuncoid* a premultifuncoid sketch such that for every $i, j \in n$ and $L \in \prod \mathfrak{Z}$

$$L_i \not\asymp \alpha_i L|_{(\text{dom } L) \setminus \{i\}} \Leftrightarrow L_j \not\asymp \alpha_j L|_{(\text{dom } L) \setminus \{j\}}. \quad (26)$$

DEFINITION 1275. Let \mathfrak{A} be an indexed family of starrish posets. The prestaroid *corresponding* to a premultifuncoid f is $[f]$ defined by the formula:

$$\text{form } [f] = \mathfrak{Z} \quad \text{and} \quad L \in \text{GR } [f] \Leftrightarrow L_i \not\asymp \langle f \rangle_i L|_{(\text{dom } L) \setminus \{i\}}.$$

PROPOSITION 1276. The prestaroid corresponding to a premultifuncoid is really a prestaroid.

PROOF. By the definition of starrish posets. □

DEFINITION 1277. I will call a *multifuncoid* a premultifuncoid to which corresponds a staroid.

DEFINITION 1278. I will call a *completary multifuncoid* a premultifuncoid to which corresponds a completary staroid.