DEFINITION 1257. We will denote the set of staroids, prestaroids, and completary staroids of a form $\mathfrak{A}$ correspondingly as $\mathsf{Strd}(\mathfrak{A})$, $\mathsf{pStrd}(\mathfrak{A})$, and $\mathsf{cStrd}(\mathfrak{A})$.

17.4. Upgrading and downgrading a set regarding a filtrator

Let fix a filtrator $(\mathfrak{A}; \mathfrak{Z})$.

DEFINITION 1258. $\downdownarrows f = f \cap \mathfrak{Z}$ for every $f \in \mathscr{P}\mathfrak{A}$ (*downgrading* f).

DEFINITION 1259. $\upuparrows f = \left\{ \frac{L \in \mathfrak{A}}{\operatorname{up} L \subseteq f} \right\}$ for every $f \in \mathscr{P}\mathfrak{Z}$ (*upgrading* f).

OBVIOUS 1260. $a \in \upuparrows f \Leftrightarrow \operatorname{up} a \subseteq f$ for every $f \in \mathscr{P}\mathfrak{Z}$ and $a \in \mathfrak{A}$.

PROPOSITION 1261. $\downdownarrows \upuparrows f = f$ if f is an upper set for every $f \in \mathscr{P}\mathfrak{Z}$.

PROOF. $\downdownarrows \upuparrows f = \upuparrows f \cap \mathfrak{Z} = \left\{ \frac{L \in \mathfrak{Z}}{\operatorname{up} L \subseteq f} \right\} = \left\{ \frac{L \in \mathfrak{Z}}{L \in f} \right\} = f \cap \mathfrak{Z} = f$. □

17.4.1. Upgrading and downgrading staroids.

Let fix a family $(\mathfrak{A}; \mathfrak{Z})$ of filtrators.

For a graph f of a staroid define $\downdownarrows f$ and $\upuparrows f$ taking the filtrator of $(\prod \mathfrak{A}; \prod \mathfrak{Z})$.

For a staroid f define:FiXme: Define for all anchored relations.

$$\operatorname{form} \downdownarrows f = \mathfrak{Z} \quad \text{and} \quad \operatorname{GR} \downdownarrows f = \downdownarrows \operatorname{GR} f;$$
$$\operatorname{form} \upuparrows f = \mathfrak{A} \quad \text{and} \quad \operatorname{GR} \upuparrows f = \upuparrows \operatorname{GR} f.$$

PROPOSITION 1262. $(\operatorname{val} \downdownarrows f)_i L = (\operatorname{val} f)_i L \cap \mathfrak{Z}_i$ for every $L \in \prod \mathfrak{Z}|_{(\operatorname{arity} f) \setminus \{i\}}$.

PROOF. $(\operatorname{val} \downdownarrows f)_i L = \left\{ \frac{X \in \mathfrak{Z}_i}{L \cup \{(i;X)\} \in \operatorname{GR} f \cap \prod \mathfrak{Z}} \right\} = \left\{ \frac{X \in \mathfrak{Z}_i}{L \cup \{(i;X)\} \in \operatorname{GR} f} \right\} = (\operatorname{val} f)_i L \cap \mathfrak{Z}_i$. □

PROPOSITION 1263. Let $(\mathfrak{A}_i; \mathfrak{Z}_i)$ be finitely join-closed filtrators with both the base and the core being join-semilattices. If f is a staroid of the form $\mathfrak{A}$, then $\downdownarrows f$ is a staroid of the form $\mathfrak{Z}$.

PROOF. Let f be a staroid.

We need to prove that $(\operatorname{val} \downdownarrows f)_i L$ is a free star. It follows from the last proposition and the fact that it is finitely join-closed. □

17.5. Principal staroids

DEFINITION 1264. The *staroid generated* by an anchored relation F is the staroid $f = \uparrow^{\mathsf{Strd}} F$ on powersets such that $\uparrow \circ L \in \operatorname{GR} f \Leftrightarrow \prod L \not\asymp F$ and $(\operatorname{form} f)_i = \mathscr{P}(\operatorname{form} F)_i$ for every $L \in \prod_{i \in \operatorname{arity} f} \mathscr{P}(\operatorname{form} F)_i$.

REMARK 1265. Below we will prove that staroid generated by an anchored relation is a staroid and moreover a completary staroid.

DEFINITION 1266. A *principal staroid* is a staroid generated by some anchored relation.

PROPOSITION 1267. Every principal staroid is a completary staroid.

PROOF. That $L \notin f$ if $L_i = \bot^{(\operatorname{form} f)_i}$ for some $i \in \operatorname{arity} f$ is obvious. It remains to prove

$$\prod (L_0 \sqcup L_1) \not\asymp F \Leftrightarrow \exists c \in \{0,1\}^{\operatorname{arity} f} : \prod_{i \in n} L_{c(i)} i \not\asymp F.$$