OBVIOUS 1243. $X \in (\operatorname{val} f)_i L \Leftrightarrow L \cup \{(i; X)\} \in \operatorname{GR} f$.

PROPOSITION 1244. $f$ can be restored knowing $\operatorname{form}(f)$ and $(\operatorname{val} f)_i$ for some $i \in \operatorname{arity} f$.

PROOF.

$$\operatorname{GR} f = \left\{ \frac{K \in \prod \operatorname{form} f}{K \in \operatorname{GR} f} \right\} = \left\{ \frac{L \cup \{(i; X)\}}{L \in \prod (\operatorname{form} f)|_{(\operatorname{arity} f) \setminus \{i\}}, X \in (\operatorname{form} f)_i, L \cup \{(i; X)\} \in \operatorname{GR} f} \right\} = \left\{ \frac{L \cup \{(i; X)\}}{L \in \prod (\operatorname{form} f)|_{(\operatorname{arity} f) \setminus \{i\}}, X \in (\operatorname{val} f)_i L} \right\}.$$

□

DEFINITION 1245. A *prestaroid* is an anchored relation $f$ between posets such that $(\operatorname{val} f)_i L$ is a free star for every $i \in \operatorname{arity} f$, $L \in \prod (\operatorname{form} f)|_{(\operatorname{arity} f) \setminus \{i\}}$.

DEFINITION 1246. A *staroid* is a prestaroid whose graph is an upper set (on the poset $\prod \operatorname{form}(f)$).

PROPOSITION 1247. If $L \in \prod \operatorname{form} f$ and $L_i = \bot^{(\operatorname{form} f)_i}$ for some $i \in \operatorname{arity} f$ then $L \notin f$ if $f$ is a prestaroid.

PROOF. Let $K = L|_{(\operatorname{arity} f) \setminus \{i\}}$. We have $\bot \notin (\operatorname{val} f)_i K$; $K \cup \{(i; 0)\} \notin f$; $L \notin f$. □

DEFINITION 1248. *Infinitary anchored relation* is such an anchored relation whose arity is infinite; *finitary anchored relation* is such an anchored relation whose arity is finite.FiXme: Move this definition up.

Next we will define *completary staroids*. First goes the general case, next simpler case for the special case of join-semilattices instead of arbitrary posets.

DEFINITION 1249. A *completary staroid* is an anchored relation between posets conforming to the formulas:

1°. $\forall K \in \prod \operatorname{form} f : (K \sqsupseteq L_0 \wedge K \sqsupseteq L_1 \Rightarrow K \in \operatorname{GR} f)$ is equivalent to $\exists c \in \{0, 1\}^n : (\lambda i \in n : L_{c(i)} i) \in \operatorname{GR} f$ for every $L_0, L_1 \in \prod \operatorname{form} f$.
2°. If $L \in \prod \operatorname{form} f$ and $L_i = \bot^{(\operatorname{form} f)_i}$ for some $i \in \operatorname{arity} f$ then $L \notin \operatorname{GR} f$.

LEMMA 1250. Every graph of completary staroid is an upper set.

PROOF. Let $f$ be a completary staroid. Let $L_0 \sqsubseteq L_1$ for some $L_0, L_1 \in \prod \operatorname{form} f$ and $L_0 \in \operatorname{GR} f$. Then taking $c = n \times \{0\}$ we get $\lambda i \in n : L_{c(i)} i = \lambda i \in n : L_0 i = L_0 \in \operatorname{GR} f$ and thus $L_1 \in \operatorname{GR} f$ because $L_1 \sqsupseteq L_0 \wedge L_1 \sqsupseteq L_1$. □

PROPOSITION 1251. A relation between posets whose form is a family of join-semilattices is a completary staroid iff both:

1°. $L_0 \sqcup L_1 \in \operatorname{GR} f \Leftrightarrow \exists c \in \{0, 1\}^n : (\lambda i \in n : L_{c(i)} i) \in \operatorname{GR} f$ for every $L_0, L_1 \in \prod \operatorname{form} f$.
2°. If $L \in \prod \operatorname{form} f$ and $L_i = \bot^{(\operatorname{form} f)_i}$ for some $i \in \operatorname{arity} f$ then $L \notin \operatorname{GR} f$.

PROOF. Let the formulas 1° and 2° hold. Then $f$ is an upper set: Let $L_0 \sqsubseteq L_1$ for some $L_0, L_1 \in \prod \operatorname{form} f$ and $L_0 \in f$. Then taking $c = n \times \{0\}$ we get $\lambda i \in n : L_{c(i)} i = \lambda i \in n : L_0 i = L_0 \in \operatorname{GR} f$ and thus $L_1 = L_0 \sqcup L_1 \in f$.

Thus to finish the proof it is enough to show that

$$L_0 \sqcup L_1 \in \operatorname{GR} f \Leftrightarrow \forall K \in \prod \operatorname{form} f : (K \sqsupseteq L_0 \wedge K \sqsupseteq L_1 \Rightarrow K \in \operatorname{GR} f)$$

under condition that $\operatorname{GR} f$ is an upper set. But this is obvious. □