PROOF. We will prove only the first, because the second is dual.

$$\begin{aligned}
&\operatorname{Cor} a = \\
&\bigsqcap^{\prod \mathfrak{Z}} \operatorname{up} a = \\
&\lambda i \in \operatorname{dom} a : \bigsqcap^{\mathfrak{Z}_i} \left\{ \frac{x_i}{x \in \operatorname{up} a} \right\} = (\operatorname{up} x \neq \emptyset \text{ taken into account}) \\
&\lambda i \in \operatorname{dom} a : \bigsqcap^{\mathfrak{Z}_i} \left\{ \frac{x}{x \in \operatorname{up} a_i} \right\} = \\
&\lambda i \in \operatorname{dom} a : \bigsqcap^{\mathfrak{Z}_i} \operatorname{up} a_i = \\
&\lambda i \in \operatorname{dom} a : \operatorname{Cor} a_i.
\end{aligned}$$

□

PROPOSITION 1231. If each $(\mathfrak{A}_i; \mathfrak{Z}_i)$ is a filtrator with (co-)separable core and each $\mathfrak{A}_i$ has a least (greatest) element, then $(\prod \mathfrak{A}; \prod \mathfrak{Z})$ is a filtrator with (co-)separable core.

PROOF. We will prove only for separable core, as co-separable core is dual.

$$\begin{aligned}
&x \asymp^{\prod \mathfrak{A}} y \Leftrightarrow \\
&(\text{used the fact that } \mathfrak{A}_i \text{ has a least element}) \\
&\forall i \in \operatorname{dom} \mathfrak{A} : x_i \asymp^{\mathfrak{A}_i} y_i \Rightarrow \\
&\forall i \in \operatorname{dom} \mathfrak{A} \exists X \in \operatorname{up} x_i : X \asymp^{\mathfrak{A}_i} y_i \Leftrightarrow \\
&\exists X \in \operatorname{up} x \forall i \in \operatorname{dom} \mathfrak{A} : X_i \asymp^{\mathfrak{A}_i} y_i \Leftrightarrow \\
&\exists X \in \operatorname{up} x : X \asymp^{\prod \mathfrak{A}} y
\end{aligned}$$

for every $x, y \in \prod \mathfrak{A}$. □

OBVIOUS 1232.

1°. If each $(\mathfrak{A}_i; \mathfrak{Z}_i)$ is a down-aligned filtrator, then $(\prod \mathfrak{A}; \prod \mathfrak{Z})$ is a down-aligned filtrator.
2°. If each $(\mathfrak{A}_i; \mathfrak{Z}_i)$ is an up-aligned filtrator, then $(\prod \mathfrak{A}; \prod \mathfrak{Z})$ is an up-aligned filtrator.

PROPOSITION 1233. If every b_i is substractive from a_i where a and b are n-indexed families of distributive lattices with least elements (where n is an index set), then $a \setminus b = \lambda i \in n : a_i \setminus b_i$.

PROOF. We need to prove $(\lambda i \in n : a_i \setminus b_i) \sqcap b = \bot$ and $a \sqcup b = b \sqcup (\lambda i \in n : a_i \setminus b_i)$. Really

$$(\lambda i \in n : a_i \setminus b_i) \sqcap b = \lambda i \in n : (a_i \setminus b_i) \sqcap b_i = \bot;$$
$$b \sqcup (\lambda i \in n : a_i \setminus b_i) = \lambda i \in n : b_i \sqcup (a_i \setminus b_i) = \lambda i \in n : b_i \sqcup a_i = a \sqcup b.$$

□

PROPOSITION 1234. If every $\mathfrak{A}_i$ is a distributive lattice, then $a \setminus^* b = \lambda i \in \operatorname{dom} \mathfrak{A} : a_i \setminus^* b_i$ for every $a, b \in \prod \mathfrak{A}$ whenever every $a_i \setminus^* b_i$ is defined.

PROOF. If some $\mathfrak{A}_i$ is empty, our statement is obvious. Let's assume $\mathfrak{A}_i \neq \emptyset$.FiXme: Should consider this special case?

We need to prove that $\lambda i \in \operatorname{dom} \mathfrak{A} : a_i \setminus^* b_i = \bigsqcap \left\{ \frac{z \in \prod \mathfrak{A}}{a \sqsubseteq b \sqcup z} \right\}$.