PROOF. We will prove only the first as the second is dual.

$$\operatorname{up} a = \left\{ \frac{c \in \prod \mathfrak{Z}}{c \sqsupseteq a} \right\} = \left\{ \frac{c \in \prod \mathfrak{Z}}{\forall i \in \operatorname{dom} a : c_i \sqsupseteq a_i} \right\} = \left\{ \frac{c \in \prod \mathfrak{Z}}{\forall i \in \operatorname{dom} a : c_i \in \operatorname{up} a_i} \right\} = \prod_{i \in \operatorname{dom} a} \operatorname{up} a_i.$$

□

PROPOSITION 1227. If every $(\mathfrak{A}_i; \mathfrak{Z}_i)$ is a filtered complete lattice filtrator, then $(\prod \mathfrak{A}; \prod \mathfrak{Z})$ is a filtered complete lattice filtrator.

PROOF. That $\prod \mathfrak{A}$ is a complete lattice is already proved above. We have for every $a \in \prod \mathfrak{A}$

$$\prod^{\prod \mathfrak{A}} \operatorname{up} a = \lambda i \in \operatorname{dom} \mathfrak{A} : \bigsqcap \left\{ \frac{x_i}{x \in \operatorname{up} a} \right\} = \lambda i \in \operatorname{dom} \mathfrak{A} : \bigsqcap \left\{ \frac{x}{x \in \operatorname{up} a_i} \right\} = \lambda i \in \operatorname{dom} \mathfrak{A} : \bigsqcap \operatorname{up} a_i = \lambda i \in \operatorname{dom} \mathfrak{A} : a_i = a.$$

□

PROPOSITION 1228. If every $(\mathfrak{A}_{i \in n}; \mathfrak{Z}_{i \in n})$ is a prefiltered complete lattice filtrator with $\operatorname{up} x \neq \emptyset$ for every $x \in \mathfrak{A}_i$ (for every $i \in n$), then $(\prod \mathfrak{A}; \prod \mathfrak{Z})$ is a prefiltered complete lattice filtrator.

PROOF. Let $a, b \in \prod \mathfrak{A}$ and $a \neq b$. Then there exists $i \in n$ such that $a_i \neq b_i$ and so $\operatorname{up} a_i \neq \operatorname{up} b_i$. Consequently $\prod_{i \in \operatorname{dom} a} \operatorname{up} a_i \neq \prod_{i \in \operatorname{dom} a} \operatorname{up} b_i$ (taken into account that $\operatorname{up} x \neq \emptyset$ for every $x \in \mathfrak{A}_i$) that is $\operatorname{up} a \neq \operatorname{up} b$. □

PROPOSITION 1229. Let every $(\mathfrak{A}_{i \in n}; \mathfrak{Z}_{i \in n})$ be a semifiltered filtrator with $\operatorname{up} x \neq \emptyset$ for every $x \in \mathfrak{A}_i$ (for every $i \in n$). Then $(\prod \mathfrak{A}; \prod \mathfrak{Z})$ is a semifiltered filtrator.FiXme: Semifiltered is the same as filtered, remove one of the two statements (which one? they are not equivalent having different theorem conditions!)

PROOF. Let every $(\mathfrak{A}_i; \mathfrak{Z}_i)$ be a semifiltered filtrator. Let $\operatorname{up} a \supseteq \operatorname{up} b$ for some $a, b \in \prod \mathfrak{A}$. Then $\prod_{i \in \operatorname{dom} a} \operatorname{up} a_i \supseteq \prod_{i \in \operatorname{dom} a} \operatorname{up} b_i$ and consequently (taking into account that $\operatorname{up} x \neq \emptyset$ for every $x \in \mathfrak{A}_i$) $\operatorname{up} a_i \supseteq \operatorname{up} b_i$ for every $i \in n$. Then $\forall i \in n : a_i \sqsubseteq b_i$ that is $a \sqsubseteq b$. □

PROPOSITION 1230. Let $(\mathfrak{A}_i; \mathfrak{Z}_i)$ be filtrators and each $\mathfrak{Z}_i$ be a complete lattice with $\operatorname{up} x \neq \emptyset$ for every $x \in \mathfrak{A}_i$ (for every $i \in n$). For $a \in \prod \mathfrak{A}$:

1°. $\operatorname{Cor} a = \lambda i \in \operatorname{dom} a : \operatorname{Cor} a_i$;
2°. $\operatorname{Cor}' a = \lambda i \in \operatorname{dom} a : \operatorname{Cor}' a_i$.