PROOF. Proposition 178. □

PROPOSITION 1222. Let $(\mathfrak{A}_{i\in n};\mathfrak{Z}_{i\in n})$ be a family of filtrators. Then $(\prod\mathfrak{A};\prod\mathfrak{Z})$ is a filtrator.

PROOF. We need to prove that $\prod\mathfrak{Z}$ is a sub-poset of $\prod\mathfrak{A}$. First $\prod\mathfrak{Z}\subseteq\prod\mathfrak{A}$ because $\mathfrak{Z}_i\subseteq\mathfrak{A}_i$ for each $i\in n$.

Let $A,B\in\prod\mathfrak{Z}$ and $A\sqsubseteq^{\prod\mathfrak{Z}}B$. Then $\forall i\in n:A_i\sqsubseteq^{\mathfrak{Z}_i}B_i$; consequently $\forall i\in n:A_i\sqsubseteq^{\mathfrak{A}_i}B_i$ that is $A\sqsubseteq^{\prod\mathfrak{A}}B$. □

PROPOSITION 1223. Let $(\mathfrak{A}_{i\in n};\mathfrak{Z}_{i\in n})$ be a family of filtrators.

1°. The filtrator $(\prod\mathfrak{A};\prod\mathfrak{Z})$ is (finitely) join-closed if every $(\mathfrak{A}_i;\mathfrak{Z}_i)$ is (finitely) join-closed.
2°. The filtrator $(\prod\mathfrak{A};\prod\mathfrak{Z})$ is (finitely) meet-closed if every $(\mathfrak{A}_i;\mathfrak{Z}_i)$ is (finitely) meet-closed.

PROOF. Let every $(\mathfrak{A}_i;\mathfrak{Z}_i)$ be finitely join-closed. Let $A,B\in\prod\mathfrak{Z}$ and $A\sqcup^{\prod\mathfrak{Z}}B$ exist. Then (by corollary 1215)

$$A\sqcup^{\prod\mathfrak{Z}}B=\lambda i\in n:A_i\sqcup^{\mathfrak{Z}_i}B_i=\lambda i\in n:A_i\sqcup^{\mathfrak{A}_i}B_i=A\sqcup^{\prod\mathfrak{A}}B.$$

Let now every $(\mathfrak{A}_i;\mathfrak{Z}_i)$ be join-closed. Let $S\in\mathscr{P}\prod\mathfrak{Z}$ and $\bigsqcup^{\prod\mathfrak{Z}}S$ exist. Then (by corollary 1215)

$$\bigsqcup^{\prod\mathfrak{Z}}S=\lambda i\in\operatorname{dom}\mathfrak{A}:\bigsqcup^{\mathfrak{Z}_i}\left\{\frac{x_i}{x\in S}\right\}=\lambda i\in\operatorname{dom}\mathfrak{A}:\bigsqcup^{\mathfrak{A}_i}\left\{\frac{x_i}{x\in S}\right\}=\bigsqcup^{\prod\mathfrak{A}}S.$$

The rest follows from symmetry. □

PROPOSITION 1224. If each $(\mathfrak{A}_i;\mathfrak{Z}_i)$ where $i\in n$ (for some index set n) is a down-aligned filtrator with separable core then $(\prod\mathfrak{A};\prod\mathfrak{Z})$ is with separable core.

PROOF. Let $a\neq b$. Then $\exists i\in n:a_i\neq b_i$. So $\exists x\in\mathfrak{Z}_i:(x\not\asymp a_i\wedge x\asymp b_i)$ (or vice versa).

Take $y=((n\setminus\{i\})\times\{0\})\cup\{(i;x)\}$. Then we have $y\not\asymp a$ and $y\asymp b$ and $y\in\mathfrak{Z}$. □

PROPOSITION 1225. Let every $\mathfrak{A}_i$ be a bounded lattice. Every $(\mathfrak{A}_i;\mathfrak{Z}_i)$ is a central filtrator iff $(\prod\mathfrak{A};\prod\mathfrak{Z})$ is a central filtrator.

PROOF. FiXme: Finish the proof.

$$\begin{aligned}x\in Z\left(\prod\mathfrak{A}\right)\Leftrightarrow\\ \exists y\in\prod\mathfrak{A}:(x\sqcap y=\bot^{\prod\mathfrak{A}}\wedge x\sqcup y=\top^{\prod\mathfrak{A}})\Leftrightarrow\\ \exists y\in\prod\mathfrak{A}\forall i\in\operatorname{dom}\mathfrak{A}:(x_i\sqcap y_i=\bot^{\mathfrak{A}_i}\wedge x_i\sqcup y_i=\top^{\mathfrak{A}_i})\Leftrightarrow\\ \forall i\in\operatorname{dom}\mathfrak{A}\exists y\in\mathfrak{A}_i:(x_i\sqcap y=\bot^{\mathfrak{A}_i}\wedge x_i\sqcup y=\top^{\mathfrak{A}_i})\Leftrightarrow\\ \forall i\in\operatorname{dom}\mathfrak{A}:x_i\in Z(\mathfrak{A}_i).\end{aligned}$$

□

PROPOSITION 1226. For every element a of a product filtrator $(\prod\mathfrak{A};\prod\mathfrak{Z})$:

1°. $\operatorname{up}a=\prod_{i\in\operatorname{dom}a}\operatorname{up}a_i$;
2°. $\operatorname{down}a=\prod_{i\in\operatorname{dom}a}\operatorname{down}a_i$.