PROOF. $\operatorname{im} f|_{\langle\mu\rangle^*\{x\}} \sqsubseteq \langle\nu\rangle^*\{y\}$; $\langle f\rangle\langle\mu\rangle^*\{x\} \sqsubseteq \langle\nu\rangle^*\{y\}$;

$$\begin{aligned}
&\nu \circ f|_{\langle\mu\rangle^*\{x\}} \sqsupseteq \\
&(\langle\nu\rangle^*\{y\} \times^{\mathsf{FCD}} \langle\nu\rangle^*\{y\}) \circ f|_{\langle\mu\rangle^*\{x\}} = \\
&\left\langle (f|_{\langle\mu\rangle^*\{x\}})^{-1} \right\rangle \langle\nu\rangle^*\{y\} \times^{\mathsf{FCD}} \langle\nu\rangle^*\{y\} = \\
&\left\langle \mathrm{id}^{\mathsf{FCD}}_{\langle\mu\rangle^*\{x\}} \circ f^{-1} \right\rangle \langle\nu\rangle^*\{y\} \times^{\mathsf{FCD}} \langle\nu\rangle^*\{y\} \sqsupseteq \\
&\left\langle \mathrm{id}^{\mathsf{FCD}}_{\langle\mu\rangle^*\{x\}} \circ f^{-1} \right\rangle \langle f\rangle\langle\mu\rangle^*\{x\} \times^{\mathsf{FCD}} \langle\nu\rangle^*\{y\} = \\
&\left\langle \mathrm{id}^{\mathsf{FCD}}_{\langle\mu\rangle^*\{x\}} \right\rangle \left\langle f^{-1} \circ f \right\rangle \langle\mu\rangle^*\{x\} \times^{\mathsf{FCD}} \langle\nu\rangle^*\{y\} \sqsupseteq \\
&\left\langle \mathrm{id}^{\mathsf{FCD}}_{\langle\mu\rangle^*\{x\}} \right\rangle \left\langle \mathrm{id}^{\mathsf{FCD}}_{\langle\mu\rangle^*\{x\}} \right\rangle \langle\mu\rangle^*\{x\} \times^{\mathsf{FCD}} \langle\nu\rangle^*\{y\} = \\
&\langle\mu\rangle^*\{x\} \times^{\mathsf{FCD}} \langle\nu\rangle^*\{y\}.
\end{aligned}$$

On the other hand, $f|_{\langle\mu\rangle^*\{x\}} \sqsubseteq \langle\mu\rangle^*\{x\} \times^{\mathsf{FCD}} \langle\nu\rangle^*\{y\}$;
$\nu \circ f|_{\langle\mu\rangle^*\{x\}} \sqsubseteq \langle\mu\rangle^*\{x\} \times^{\mathsf{FCD}} \langle\nu\rangle\langle\nu\rangle^*\{y\} \sqsubseteq \langle\mu\rangle^*\{x\} \times^{\mathsf{FCD}} \langle\nu\rangle^*\{y\}$.
So $\nu \circ f|_{\langle\mu\rangle^*\{x\}} = \langle\mu\rangle^*\{x\} \times^{\mathsf{FCD}} \langle\nu\rangle^*\{y\}$.
$\operatorname{xlim}_x f = \left\{ \frac{\nu \circ f|_{\langle\mu\rangle^*\{x\}} \circ \uparrow r}{r \in G} \right\} = \left\{ \frac{(\langle\mu\rangle^*\{x\} \times^{\mathsf{FCD}} \langle\nu\rangle^*\{y\}) \circ \uparrow r}{r \in G} \right\} = \tau(y)$. □

COROLLARY 1195. If $\lim^{\nu}_{\langle\mu\rangle^*\{x\}} f = y$ then $\operatorname{xlim}_x f = \tau(y)$.

We have injective τ if $\langle\nu\rangle^*\{y_1\} \sqcap \langle\nu\rangle^*\{y_2\} = 0^{\mathfrak{F}(\operatorname{Ob}\mu)}$ for every distinct $y_1, y_2 \in \operatorname{Ob}\nu$ that is if ν is T_2-separable.