

Reversely $\langle \mu \rangle^* \{x\} \times^{\text{FCD}} \langle \nu \rangle^* \{y\} = (\langle \mu \rangle^* \{x\} \times^{\text{FCD}} \langle \nu \rangle^* \{y\}) \circ \uparrow e$ where e is the identify element of G . \square

PROPOSITION 1191. $\tau(y) = \text{xlim}(\langle \mu \rangle^* \{x\} \times^{\text{FCD}} \uparrow^{\text{Base}(\text{Ob } \nu)} \{y\})$ (for every x). Informally: Every $\tau(y)$ is a generalized limit of a constant funcoid.

PROOF.

$$\begin{aligned} \text{xlim}(\langle \mu \rangle^* \{x\} \times^{\text{FCD}} \uparrow^{\text{Base}(\text{Ob } \nu)} \{y\}) &= \\ \left\{ \frac{\nu \circ (\langle \mu \rangle^* \{x\} \times^{\text{FCD}} \uparrow^{\text{Base}(\text{Ob } \nu)} \{y\}) \circ \uparrow r}{r \in G} \right\} &= \\ \left\{ \frac{(\langle \mu \rangle^* \{x\} \times^{\text{FCD}} \langle \nu \rangle^* \{y\}) \circ \uparrow r}{r \in G} \right\} &= \tau(y). \end{aligned}$$

\square

THEOREM 1192. If $f|_{\langle \mu \rangle^* \{x\}} \in \text{C}(\mu; \nu)$ and $\langle \mu \rangle^* \{x\} \sqsupseteq \uparrow^{\text{Ob } \mu} \{x\}$ then $\text{xlim}_x f = \tau(fx)$.

PROOF. $f|_{\langle \mu \rangle^* \{x\}} \circ \mu \sqsubseteq \nu \circ f|_{\langle \mu \rangle^* \{x\}} \sqsubseteq \nu \circ f$; thus $\langle f \rangle \langle \mu \rangle^* \{x\} \sqsubseteq \langle \nu \rangle \langle f \rangle^* \{x\}$; consequently we have

$$\begin{aligned} \nu \sqsupseteq \langle \nu \rangle \langle f \rangle^* \{x\} \times^{\text{FCD}} \langle \nu \rangle \langle f \rangle^* \{x\} &\sqsupseteq \langle f \rangle \langle \mu \rangle^* \{x\} \times^{\text{FCD}} \langle \nu \rangle \langle f \rangle^* \{x\}. \\ \nu \circ f|_{\langle \mu \rangle^* \{x\}} &\sqsupseteq \\ \langle f \rangle \langle \mu \rangle^* \{x\} \times^{\text{FCD}} \langle \nu \rangle \langle f \rangle^* \{x\} \circ f|_{\langle \mu \rangle^* \{x\}} &= \\ (f|_{\langle \mu \rangle^* \{x\}})^{-1} \langle f \rangle \langle \mu \rangle^* \{x\} \times^{\text{FCD}} \langle \nu \rangle \langle f \rangle^* \{x\} &\sqsupseteq \\ \left\langle \text{id}_{\text{dom } f|_{\langle \mu \rangle^* \{x\}}}^{\text{FCD}} \right\rangle \langle \mu \rangle^* \{x\} \times^{\text{FCD}} \langle \nu \rangle \langle f \rangle^* \{x\} &\sqsupseteq \\ \text{dom } f|_{\langle \mu \rangle^* \{x\}} \times^{\text{FCD}} \langle \nu \rangle \langle f \rangle^* \{x\} &= \\ \langle \mu \rangle^* \{x\} \times^{\text{FCD}} \langle \nu \rangle \langle f \rangle^* \{x\}. & \end{aligned}$$

$$\text{im}(\nu \circ f|_{\langle \mu \rangle^* \{x\}}) = \langle \nu \rangle \langle f \rangle^* \{x\};$$

$$\begin{aligned} \nu \circ f|_{\langle \mu \rangle^* \{x\}} &\sqsubseteq \\ \langle \mu \rangle^* \{x\} \times^{\text{FCD}} \text{im}(\nu \circ f|_{\langle \mu \rangle^* \{x\}}) &= \\ \langle \mu \rangle^* \{x\} \times^{\text{FCD}} \langle \nu \rangle \langle f \rangle^* \{x\}. & \end{aligned}$$

So $\nu \circ f|_{\langle \mu \rangle^* \{x\}} = \langle \mu \rangle^* \{x\} \times^{\text{FCD}} \langle \nu \rangle \langle f \rangle^* \{x\}$.

$$\text{Thus } \text{xlim}_x f = \left\{ \frac{(\langle \mu \rangle^* \{x\} \times^{\text{FCD}} \langle \nu \rangle \langle f \rangle^* \{x\}) \circ \uparrow r}{r \in G} \right\} = \tau(fx). \quad \square$$

REMARK 1193. Without the requirement of $\langle \mu \rangle^* \{x\} \sqsupseteq \uparrow^{\text{Ob } \mu} \{x\}$ the last theorem would not work in the case of removable singularity.

THEOREM 1194. Let $\nu \sqsubseteq \nu \circ \nu$. If $f|_{\langle \mu \rangle^* \{x\}} \xrightarrow{\nu} \uparrow^{\text{Ob } \mu} \{y\}$ then $\text{xlim}_x f = \tau(y)$.