16.2. Relationships between convergence and continuity

Lemma 1174. Let μ, ν be endofuncoids, $f \in \mathsf{FCD}(\operatorname{Ob}\mu; \operatorname{Ob}\nu)$, $\mathcal{A} \in \mathfrak{F}(\operatorname{Ob}\mu)$, $\operatorname{Src} f = \operatorname{Ob}\mu$, $\operatorname{Dst} f = \operatorname{Ob}\nu$. If $f \in \mathrm{C}(\mu|_{\mathcal{A}}; \nu)$ then

$$f|_{\langle\mu\rangle\mathcal{A}} \xrightarrow{\mu} \langle f\rangle\mathcal{A} \Leftrightarrow \langle f \circ \mu|_{\mathcal{A}}\rangle\mathcal{A} \sqsubseteq \langle \nu \circ f\rangle\mathcal{A}.$$

Proof.

$$f|_{\langle\mu\rangle\mathcal{A}} \xrightarrow{\mu} \langle f\rangle\mathcal{A} \Leftrightarrow \operatorname{im} f|_{\langle\mu\rangle\mathcal{A}} \sqsubseteq \langle\nu\rangle\langle f\rangle\mathcal{A} \Leftrightarrow \\ \langle f\rangle\langle\mu\rangle\mathcal{A} \sqsubseteq \langle\nu\rangle\langle f\rangle\mathcal{A} \Leftrightarrow \langle f \circ \mu\rangle\mathcal{A} \sqsubseteq \langle \nu \circ f\rangle\mathcal{A} \Leftrightarrow \langle f \circ \mu|_{\mathcal{A}}\rangle\mathcal{A} \sqsubseteq \langle \nu \circ f\rangle\mathcal{A}.$$

□

Theorem 1175. Let μ, ν be endofuncoids, $f \in \mathsf{FCD}(\operatorname{Ob}\mu; \operatorname{Ob}\nu)$, $\mathcal{A} \in \mathfrak{F}(\operatorname{Ob}\mu)$, $\operatorname{Src} f = \operatorname{Ob}\mu$, $\operatorname{Dst} f = \operatorname{Ob}\nu$. If $f \in \mathrm{C}(\mu|_{\mathcal{A}}; \nu)$ then $f|_{\langle\mu\rangle\mathcal{A}} \xrightarrow{\nu} \langle f\rangle\mathcal{A}$.

Proof.

$$f|_{\langle\mu\rangle\mathcal{A}} \xrightarrow{\nu} \langle f\rangle\mathcal{A} \Leftrightarrow \text{(by the lemma)} \Leftrightarrow \langle f \circ \mu|_{\mathcal{A}}\rangle\mathcal{A} \sqsubseteq \langle \nu \circ f\rangle\mathcal{A} \Leftarrow \\ f \circ \mu|_{\mathcal{A}} \sqsubseteq \nu \circ f \Leftrightarrow f \in \mathrm{C}(\mu|_{\mathcal{A}}; \nu).$$

□

Corollary 1176. Let μ, ν be endofuncoids, $f \in \mathsf{FCD}(\operatorname{Ob}\mu; \operatorname{Ob}\nu)$, $\mathcal{A} \in \mathfrak{F}(\operatorname{Ob}\mu)$, $\operatorname{Src} f = \operatorname{Ob}\mu$, $\operatorname{Dst} f = \operatorname{Ob}\nu$. If $f \in \mathrm{C}(\mu; \nu)$ then $f|_{\langle\mu\rangle\mathcal{A}} \xrightarrow{\nu} \langle f\rangle\mathcal{A}$.

Theorem 1177. Let μ, ν be endofuncoids, $f \in \mathsf{FCD}(\operatorname{Ob}\mu; \operatorname{Ob}\nu)$, $\mathcal{A} \in \mathfrak{F}(\operatorname{Ob}\mu)$ be an ultrafilter, $\operatorname{Src} f = \operatorname{Ob}\mu$, $\operatorname{Dst} f = \operatorname{Ob}\nu$. $f \in \mathrm{C}(\mu|_{\mathcal{A}}; \nu)$ iff $f|_{\langle\mu\rangle\mathcal{A}} \xrightarrow{\nu} \langle f\rangle\mathcal{A}$.

Proof.

$$f|_{\langle\mu\rangle\mathcal{A}} \xrightarrow{\nu} \langle f\rangle\mathcal{A} \Leftrightarrow \text{(by the lemma)} \Leftrightarrow \langle f \circ \mu|_{\mathcal{A}}\rangle\mathcal{A} \sqsubseteq \langle \nu \circ f\rangle\mathcal{A} \Leftrightarrow \\ \text{(used the fact that } \mathcal{A} \text{ is an ultrafilter)} \\ f \circ \mu|_{\mathcal{A}} \sqsubseteq \nu \circ f|_{\mathcal{A}} \Leftrightarrow f \circ \mu|_{\mathcal{A}} \sqsubseteq \nu \circ f \Leftrightarrow f \in \mathrm{C}(\mu|_{\mathcal{A}}; \nu).$$

□

16.3. Limit

Definition 1178. $\lim^{\mu} f = a$ iff $f \xrightarrow{\mu} \uparrow^{\operatorname{Src}\mu} \{a\}$ for a T_2-separable funcoid μ and a non-empty funcoid f such that $\operatorname{Dst} f = \operatorname{Dst}\mu$.

It is defined correctly, that is f has no more than one limit.

Proof. Let $\lim^{\mu} f = a$ and $\lim^{\mu} f = b$. Then $\operatorname{im} f \sqsubseteq \langle\mu\rangle^{*}\{a\}$ and $\operatorname{im} f \sqsubseteq \langle\mu\rangle^{*}\{b\}$.

Because $f \neq \bot^{\mathsf{FCD}(\operatorname{Src} f; \operatorname{Dst} f)}$ we have $\operatorname{im} f \neq \bot^{\mathfrak{F}(\operatorname{Dst} f)}$; $\langle\mu\rangle^{*}\{a\} \sqcap \langle\mu\rangle^{*}\{b\} \neq \bot^{\mathfrak{F}(\operatorname{Dst} f)}$; $\uparrow^{\operatorname{Src}\mu} \{b\} \sqcap \langle\mu^{-1}\rangle\langle\mu\rangle^{*}\{a\} \neq \bot^{\mathfrak{F}(\operatorname{Src}\mu)}$; $\uparrow^{\operatorname{Src}\mu} \{b\} \sqcap \langle\mu^{-1} \circ \mu\rangle \uparrow^{\operatorname{Src}\mu} \{a\} \neq \bot^{\mathfrak{F}(\operatorname{Src}\mu)}$; $\{a\} \left[\mu^{-1} \circ \mu\right] \{b\}$. Because μ is T_2-separable we have $a = b$. □

Definition 1179. $\lim^{\mu}_{\mathcal{B}} f = \lim^{\mu}(f|_{\mathcal{B}})$.

Remark 1180. We can also in an obvious way define limit of a reloid.

16.4. Generalized limit

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