CHAPTER 16

Convergence of funcoids

16.1. Convergence

The following generalizes the well-known notion of a filter convergent to a point or to a set:FiXme: Can we generalize this for pointfree funcoids?

DEFINITION 1168. A filter $\mathcal{F} \in \mathfrak{F}(\operatorname{Dst}\mu)$ *converges* to a filter $\mathcal{A} \in \mathfrak{F}(\operatorname{Src}\mu)$ regarding a funcoid μ ($\mathcal{F} \xrightarrow{\mu} \mathcal{A}$) iff $\mathcal{F} \sqsubseteq \langle\mu\rangle\mathcal{A}$.

DEFINITION 1169. A funcoid f *converges* to a filter $\mathcal{A} \in \mathfrak{F}(\operatorname{Src}\mu)$ regarding a funcoid μ where $\operatorname{Dst} f = \operatorname{Dst}\mu$ (denoted $f \xrightarrow{\mu} \mathcal{A}$) iff $\operatorname{im} f \sqsubseteq \langle\mu\rangle\mathcal{A}$ that is iff $\operatorname{im} f \xrightarrow{\mu} \mathcal{A}$.

DEFINITION 1170. A funcoid f *converges* to a filter $\mathcal{A} \in \mathfrak{F}(\operatorname{Src}\mu)$ on a filter $\mathcal{B} \in \mathfrak{F}(\operatorname{Src} f)$ regarding a funcoid μ where $\operatorname{Dst} f = \operatorname{Dst}\mu$ iff $f|_{\mathcal{B}} \xrightarrow{\mu} \mathcal{A}$.

REMARK 1171. We can define also convergence for a reloid f: $f \xrightarrow{\mu} \mathcal{A} \Leftrightarrow \operatorname{im} f \sqsubseteq \langle\mu\rangle\mathcal{A}$ or what is the same $f \xrightarrow{\mu} \mathcal{A} \Leftrightarrow (\mathsf{FCD})f \xrightarrow{\mu} \mathcal{A}$.

THEOREM 1172. Let f, g be funcoids, μ, ν be endofuncoids, $\operatorname{Dst} f = \operatorname{Src} g = \operatorname{Ob}\mu$, $\operatorname{Dst} g = \operatorname{Ob}\nu$, $\mathcal{A} \in \mathfrak{F}(\operatorname{Ob}\mu)$. If $f \xrightarrow{\mu} \mathcal{A}$,

$$g|_{\langle\mu\rangle\mathcal{A}} \in \mathrm{C}(\mu \sqcap (\langle\mu\rangle\mathcal{A} \times^{\mathsf{FCD}} \langle\mu\rangle\mathcal{A}); \nu),$$

and $\langle\mu\rangle\mathcal{A} \sqsupseteq \mathcal{A}$, then $g \circ f \xrightarrow{\nu} \langle g\rangle\mathcal{A}$.

PROOF.

$$\operatorname{im} f \sqsubseteq \langle\mu\rangle\mathcal{A};$$
$$\langle g\rangle \operatorname{im} f \sqsubseteq \langle g\rangle\langle\mu\rangle\mathcal{A};$$
$$\operatorname{im}(g \circ f) \sqsubseteq \left\langle g|_{\langle\mu\rangle\mathcal{A}}\right\rangle\langle\mu\rangle\mathcal{A};$$
$$\operatorname{im}(g \circ f) \sqsubseteq \left\langle g|_{\langle\mu\rangle\mathcal{A}}\right\rangle\left\langle\mu \sqcap (\langle\mu\rangle\mathcal{A} \times^{\mathsf{FCD}} \langle\mu\rangle\mathcal{A})\right\rangle\mathcal{A};$$
$$\operatorname{im}(g \circ f) \sqsubseteq \left\langle g|_{\langle\mu\rangle\mathcal{A}} \circ (\mu \sqcap (\langle\mu\rangle\mathcal{A} \times^{\mathsf{FCD}} \langle\mu\rangle\mathcal{A}))\right\rangle\mathcal{A};$$
$$\operatorname{im}(g \circ f) \sqsubseteq \left\langle \nu \circ g|_{\langle\mu\rangle\mathcal{A}}\right\rangle\mathcal{A};$$
$$\operatorname{im}(g \circ f) \sqsubseteq \langle \nu \circ g\rangle\mathcal{A};$$
$$g \circ f \xrightarrow{\nu} \langle g\rangle\mathcal{A}.$$

□

COROLLARY 1173. Let f, g be funcoids, μ, ν be endofuncoids, $\operatorname{Dst} f = \operatorname{Src} g = \operatorname{Ob}\mu$, $\operatorname{Dst} g = \operatorname{Ob}\nu$, $\mathcal{A} \in \mathfrak{F}(\operatorname{Ob}\mu)$. If $f \xrightarrow{\mu} \mathcal{A}$, $g \in \mathrm{C}(\mu; \nu)$, and $\langle\mu\rangle\mathcal{A} \sqsupseteq \mathcal{A}$ then $g \circ f \xrightarrow{\nu} \langle g\rangle\mathcal{A}$.

PROOF. From the last theorem and theorem 873. □