PROPOSITION 1164. If S is a set of elements closed regarding a complete pointfree funcoid f with separable destination which is a complete lattice, then the element $\bigsqcup S$ is also closed regarding our funcoid.

PROOF. $\langle f\rangle \bigsqcup S = \bigsqcup \langle\langle f\rangle\rangle^* S \sqsubseteq \bigsqcup S$. □

15.15. Connectedness regarding a pointfree funcoid

Let $\mathfrak{A}$ be a poset with least element. Let $\mu \in \mathsf{FCD}(\mathfrak{A}; \mathfrak{A})$.FiXme: No necessity for least element.

DEFINITION 1165. An element $a \in \mathfrak{A}$ is called *connected* regarding a pointfree funcoid μ over $\mathfrak{A}$ when

$$\forall x, y \in \mathfrak{A} \setminus \{\bot^{\mathfrak{A}}\} : (x \sqcup y = a \Rightarrow x\ [\mu]\ y).$$

PROPOSITION 1166. Let $(\mathfrak{A}; \mathfrak{Z})$ be a co-separable filtrator with join-closed core. An $A \in \mathfrak{Z}$ is connected regarding a funcoid μ iff

$$\forall X, Y \in \mathfrak{Z} \setminus \{\bot^{\mathfrak{Z}}\} : (X \sqcup^{\mathfrak{Z}} Y = A \Rightarrow X\ [\mu]\ Y).$$

PROOF.

⇒. Obvious.
⇐. Follows from co-separability.

□

OBVIOUS 1167. For $\mathfrak{A}$ being a set of filters over a boolean lattice, an element $a \in \mathfrak{A}$ is connected regarding a pointfree funcoid μ iff it is connected regarding the funcoid $\mu \sqcap (a \times^{\mathsf{FCD}} a)$.