1°⇒2°. For every $S \in \mathscr{P}\mathfrak{A}$, $J \in \mathfrak{Z}_1$ we have $\langle f^{-1} \rangle J \in \mathfrak{Z}_0$, consequently

$$\forall S \in \mathscr{P}\mathfrak{A}, J \in \mathfrak{Z}_1 : \left(\bigsqcup^{\mathfrak{A}} S \not\asymp \langle f^{-1} \rangle J \Rightarrow \exists \mathcal{I} \in S : \mathcal{I} \not\asymp \langle f^{-1} \rangle J \right).$$

From this follows 2°.

2°⇒4°. Let $\langle f \rangle \bigsqcup^{\mathfrak{Z}_0} S$ and $\bigsqcup^{\mathfrak{B}} \langle \langle f \rangle \rangle^* S$ be defined. We have $\langle f \rangle \bigsqcup^{\mathfrak{A}} S = \langle f \rangle \bigsqcup^{\mathfrak{Z}_0} S$.

$$J \sqcap^{\mathfrak{B}} \langle f \rangle \bigsqcup^{\mathfrak{A}} S \neq \bot^{\mathfrak{B}} \Leftrightarrow$$
$$\bigsqcup^{\mathfrak{A}} S \; [f] \; J \Leftrightarrow$$
$$\exists \mathcal{I} \in S : \mathcal{I} \; [f] \; J \Leftrightarrow$$
$$\exists \mathcal{I} \in S : J \sqcap^{\mathfrak{B}} \langle f \rangle \mathcal{I} \neq \bot^{\mathfrak{B}} \Leftrightarrow$$
$$J \sqcap^{\mathfrak{B}} \bigsqcup^{\mathfrak{B}} \langle \langle f \rangle \rangle^* S \neq \bot^{\mathfrak{B}}$$

(used theorem 320). Thus $\langle f \rangle \bigsqcup^{\mathfrak{A}} S = \bigsqcup^{\mathfrak{B}} \langle \langle f \rangle \rangle^* S$ by star-separability of $(\mathfrak{B}; \mathfrak{Z}_1)$.

5°⇒3°. Let $\langle f \rangle \bigsqcup^{\mathfrak{Z}_0} S$ be defined. Then $\bigsqcup^{\mathfrak{B}} \langle \langle f \rangle \rangle^* S$ is also defined because $\langle f \rangle \bigsqcup^{\mathfrak{Z}_0} S = \bigsqcup^{\mathfrak{B}} \langle \langle f \rangle \rangle^* S$. Then

$$\bigsqcup^{\mathfrak{Z}_0} S \; [f] \; J \Leftrightarrow J \sqcap^{\mathfrak{B}} \langle f \rangle \bigsqcup^{\mathfrak{Z}_0} S \neq \bot^{\mathfrak{B}} \Leftrightarrow J \sqcap^{\mathfrak{B}} \bigsqcup^{\mathfrak{B}} \langle \langle f \rangle \rangle^* S \neq \bot^{\mathfrak{B}}$$

what by theorem 320 is equivalent to $\exists I \in S : J \sqcap^{\mathfrak{B}} \langle f \rangle I \neq \bot^{\mathfrak{B}}$ that is $\exists I \in S : I \; [f] \; J$.

2°⇒3°, 4°⇒5°. By join-closedness of the core of $(\mathfrak{A}; \mathfrak{Z}_0)$.

□

THEOREM 1149. Let $(\mathfrak{A}; \mathfrak{Z}_0)$ and $(\mathfrak{B}; \mathfrak{Z}_1)$ be primary filtrators over boolean lattices. If R is a set of co-complete pointfree funcoids in $\mathsf{FCD}(\mathfrak{A}; \mathfrak{B})$ then $\bigsqcup R$ is a co-complete pointfree funcoid.

PROOF. Let R be a set of co-complete pointfree funcoids. Then for every $X \in \mathfrak{Z}_0$

$$\left\langle \bigsqcup R \right\rangle X = \bigsqcup^{\mathfrak{B}} \left\{ \frac{\langle f \rangle X}{f \in R} \right\} = \bigsqcup^{\mathfrak{Z}_1} \left\{ \frac{\langle f \rangle X}{f \in R} \right\} \in \mathfrak{Z}_1$$

(used the theorem 1089 and corollary 363). □

Let $\mathfrak{A}$ and $\mathfrak{B}$ be posets with least elements. I will denote $\mathsf{ComplFCD}(\mathfrak{A}; \mathfrak{B})$ and $\mathsf{CoComplFCD}(\mathfrak{A}; \mathfrak{B})$ the sets of complete and co-complete funcoids correspondingly from a poset $\mathfrak{A}$ to a poset $\mathfrak{B}$.

PROPOSITION 1150.

1°. Let $f \in \mathsf{ComplFCD}(\mathfrak{A}; \mathfrak{B})$ and $g \in \mathsf{ComplFCD}(\mathfrak{B}; \mathfrak{C})$ where $\mathfrak{A}$ and $\mathfrak{C}$ are posets with least elements and $\mathfrak{B}$ is a complete lattice. Then $g \circ f \in \mathsf{ComplFCD}(\mathfrak{A}; \mathfrak{C})$.

2°. Let $f \in \mathsf{CoComplFCD}(\mathfrak{A}; \mathfrak{B})$ and $g \in \mathsf{CoComplFCD}(\mathfrak{B}; \mathfrak{C})$ where $\mathfrak{A}$, $\mathfrak{B}$ and $\mathfrak{C}$ are posets with least elements and $(\mathfrak{A}; \mathfrak{Z}_0)$, $(\mathfrak{B}; \mathfrak{Z}_1)$, $(\mathfrak{C}; \mathfrak{Z}_2)$ are filtrators. Then $g \circ f \in \mathsf{CoComplFCD}(\mathfrak{A}; \mathfrak{C})$.

PROOF.

1°. Let $\bigsqcup S$ and $\bigsqcup \langle \langle g \circ f \rangle \rangle^* S$ be defined. ThenFiXme: Is $\bigsqcup \langle \langle f \rangle \rangle^* S$ defined?

$$\langle g \circ f \rangle \bigsqcup S = \langle g \rangle \langle f \rangle \bigsqcup S = \langle g \rangle \bigsqcup \langle \langle f \rangle \rangle^* S = \bigsqcup \langle \langle g \rangle \rangle^* \langle \langle f \rangle \rangle^* S = \bigsqcup \langle \langle g \circ f \rangle \rangle^* S.$$