PROOF.

$$(a \times^{\mathsf{FCD}} b) \sqcap (f \sqcup g) \neq \emptyset \Leftrightarrow a\ [f \sqcup g]\ b \Leftrightarrow a\ [f]\ b \vee a\ [g]\ b \Leftrightarrow (a \times^{\mathsf{FCD}} b) \sqcap f \neq \bot^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} \vee (a \times^{\mathsf{FCD}} b) \sqcap g \neq \bot^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})}$$

for every $a \in \operatorname{atoms}^{\mathfrak{A}}$ and $b \in \operatorname{atoms}^{\mathfrak{B}}$ (used the corollary 1125 and theorem 1091). □

THEOREM 1139. Let $(\mathfrak{A}; \mathfrak{Z}_0)$ and $(\mathfrak{B}; \mathfrak{Z}_1)$ be primary filtrators over boolean lattices. For every $f, g, h \in \mathsf{FCD}(\mathfrak{A}; \mathfrak{B})$, $R \in \mathscr{P}\mathsf{FCD}(\mathfrak{A}; \mathfrak{B})$:

1°. $f \sqcap (g \sqcup h) = (f \sqcap g) \sqcup (f \sqcap h)$;
2°. $f \sqcup \bigsqcap R = \bigsqcap \langle f \sqcup \rangle^* R$.

PROOF. We will take into account that the lattice of pointfree funcoids is an atomistic lattice (corollary 1137). □

1°.

$$\begin{aligned}
&\operatorname{atoms}(f \sqcap (g \sqcup h)) = \\
&\operatorname{atoms} f \cap \operatorname{atoms}(g \sqcup h) = \\
&\operatorname{atoms} f \cap (\operatorname{atoms} g \cup \operatorname{atoms} h) = \\
&(\operatorname{atoms} f \cap \operatorname{atoms} g) \cup (\operatorname{atoms} f \cap \operatorname{atoms} h) = \\
&\operatorname{atoms}(f \sqcap g) \cup \operatorname{atoms}(f \sqcap h) = \\
&\operatorname{atoms}((f \sqcap g) \sqcup (f \sqcap h)).
\end{aligned}$$

2°.

$$\begin{aligned}
&\operatorname{atoms}\left(f \sqcup \bigsqcap R\right) = \\
&\operatorname{atoms} f \cup \operatorname{atoms} \bigsqcap R = \\
&\operatorname{atoms} f \cup \bigcap \langle \operatorname{atoms} \rangle^* R = \\
&\bigcap \langle (\operatorname{atoms} f) \cup \rangle^* \langle \operatorname{atoms} \rangle^* R = \quad \text{(use the following equality)} \\
&\bigcap \langle \operatorname{atoms} \rangle^* \langle f \sqcup \rangle^* R = \\
&\operatorname{atoms} \bigsqcap \langle f \sqcup \rangle^* R.
\end{aligned}$$

$$\begin{aligned}
&\langle (\operatorname{atoms} f) \cup \rangle^* \langle \operatorname{atoms} \rangle^* R = \\
&\left\{ \frac{(\operatorname{atoms} f) \cup A}{A \in \langle \operatorname{atoms} \rangle^* R} \right\} = \\
&\left\{ \frac{(\operatorname{atoms} f) \cup A}{\exists C \in R : A = \operatorname{atoms} C} \right\} = \\
&\left\{ \frac{(\operatorname{atoms} f) \cup (\operatorname{atoms} C)}{C \in R} \right\} = \\
&\left\{ \frac{\operatorname{atoms}(f \sqcup C)}{C \in R} \right\} = \\
&\left\{ \frac{\operatorname{atoms} B}{\exists C \in R : B = f \sqcup C} \right\} = \\
&\left\{ \frac{\operatorname{atoms} B}{B \in \langle f \sqcup \rangle^* C} \right\} = \\
&\langle \operatorname{atoms} \rangle^* \langle f \sqcup \rangle^* R.
\end{aligned}$$