$\Leftarrow$. Let $a \in \operatorname{atoms}^{\mathfrak{A}}$, $b \in \operatorname{atoms}^{\mathfrak{B}}$, $f \in \mathsf{FCD}(\mathfrak{A};\mathfrak{B})$. If $b \asymp^{\mathfrak{B}} \langle f\rangle a$ then $\neg(a\ [f]\ b)$, $f \sqcap (a \times^{\mathsf{FCD}} b) = \bot^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})}$ (because $\mathfrak{A}$ and $\mathfrak{B}$ are bounded meet-semilattices); if $b \sqsubseteq \langle f\rangle a$ then $\forall \mathcal{X} \in \mathfrak{A} : (\mathcal{X} \not\asymp a \Rightarrow \langle f\rangle \mathcal{X} \sqsupseteq b)$, $f \sqsupseteq a \times^{\mathsf{FCD}} b$. Consequently $f \sqcap (a \times^{\mathsf{FCD}} b) = \bot^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} \vee f \sqsupseteq a \times^{\mathsf{FCD}} b$; that is $a \times^{\mathsf{FCD}} b$ is an atomic pointfree funcoid.

□

THEOREM 1133. Let $\mathfrak{A}$, $\mathfrak{B}$ be sets of filters over boolean lattices. Then $\mathsf{FCD}(\mathfrak{A};\mathfrak{B})$ is atomic.

PROOF. Let $f \in \mathsf{FCD}(\mathfrak{A};\mathfrak{B})$ and $f \neq \bot^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})}$. Then $\operatorname{dom} f \neq \bot^{\mathfrak{A}}$, thus exists $a \in \operatorname{atoms} \operatorname{dom} f$. So $\langle f\rangle a \neq \bot^{\mathfrak{B}}$ thus exists $b \in \operatorname{atoms}\langle f\rangle a$. Finally the atomic pointfree funcoid $a \times^{\mathsf{FCD}} b \sqsubseteq f$. □

THEOREM 1134. Let $\mathfrak{A}$, $\mathfrak{B}$ be sets of filters over boolean lattices. Then the poset $\mathsf{FCD}(\mathfrak{A};\mathfrak{B})$ is separable.

PROOF. Let $f, g \in \mathsf{FCD}(\mathfrak{A};\mathfrak{B})$, $f \sqsubset g$. Then taking into account the theorem 632 exists $a \in \operatorname{atoms}^{\mathfrak{A}}$ such that $\langle f\rangle a \sqsubset \langle g\rangle a$. By corollary 405 $\mathfrak{B}$ is atomically separable. So exists $b \in \operatorname{atoms}^{\mathfrak{B}}$ such that $\langle f\rangle a \sqcap b = \bot^{\mathfrak{B}}$ and $b \sqsubseteq \langle g\rangle a$. For every $x \in \operatorname{atoms}^{\mathfrak{A}}$

$$\langle f\rangle a \sqcap \langle a \times^{\mathsf{FCD}} b\rangle a = \langle f\rangle a \sqcap b = \bot^{\mathfrak{B}},$$
$$x \neq a \Rightarrow \langle f\rangle x \sqcap \langle a \times^{\mathsf{FCD}} b\rangle x = \langle f\rangle x \sqcap \bot^{\mathfrak{B}} = \bot^{\mathfrak{B}}.$$

Thus $\langle f\rangle x \sqcap \langle a \times^{\mathsf{FCD}} b\rangle x = \bot^{\mathfrak{B}}$ and consequently $f \sqcap (a \times^{\mathsf{FCD}} b) = \bot^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})}$.

$$\langle a \times^{\mathsf{FCD}} b\rangle a = b \sqsubseteq \langle g\rangle a,$$
$$x \neq a \Rightarrow \langle a \times^{\mathsf{FCD}} b\rangle x = \bot^{\mathfrak{B}} \sqsubseteq \langle g\rangle x.$$

Thus $\langle a \times^{\mathsf{FCD}} b\rangle x \sqsubseteq \langle g\rangle x$ and consequently $a \times^{\mathsf{FCD}} b \sqsubseteq g$.

So the lattice of pointfree funcoids is separable by the theorem 173. □

COROLLARY 1135. FiXme: Wrong theorem's label.Let $\mathfrak{A}$, $\mathfrak{B}$ be sets of filters over boolean lattices. The poset $\mathsf{FCD}(\mathfrak{A};\mathfrak{B})$ is:

1°. separable;
2°. atomically separable;
3°. conforming to Wallman's disjunction property.

PROOF. By the theorem 180. □

REMARK 1136. For more ways to characterize (atomic) separability of the lattice of pointfree funcoids see subsections Separation subsets and full stars and Atomically Separable Lattices.

COROLLARY 1137. Let $(\mathfrak{A};\mathfrak{Z}_0)$ and $(\mathfrak{B};\mathfrak{Z}_1)$ be primary filtrators over boolean lattices. The poset $\mathsf{FCD}(\mathfrak{A};\mathfrak{B})$ is an atomistic lattice.

PROOF. FiXme: Is there a shorter proof just referring to a ready proposition?By the corollary 1090 $\mathsf{FCD}(\mathfrak{A};\mathfrak{B})$ is a complete lattice. Let $f \in \mathsf{FCD}(\mathfrak{A};\mathfrak{B})$. Suppose contrary to the statement to be proved that $\bigsqcup \operatorname{atoms} f \sqsubset f$. Then there exists $a \in \operatorname{atoms} f$ such that $a \sqcap \bigsqcup \operatorname{atoms} f = \bot^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})}$ what is impossible. □

PROPOSITION 1138. Let $\mathfrak{A}$, $\mathfrak{B}$ be sets of filters over boolean lattices. $\operatorname{atoms}(f \sqcup g) = \operatorname{atoms} f \cup \operatorname{atoms} g$ for every $f, g \in \mathsf{FCD}(\mathfrak{A};\mathfrak{B})$.