Let's prove the reverse of (18):

$$\begin{aligned}\bigsqcap\left\langle\bigsqcup\circ\langle\alpha\rangle^*\circ\operatorname{atoms}^{\mathfrak{A}}\right\rangle^*\operatorname{up}^{(\mathfrak{A};\,\mathfrak{Z}_0)}a&=\\ \bigsqcap\left\langle\bigsqcup\circ\langle\alpha\rangle^*\right\rangle^*\langle\operatorname{atoms}^{\mathfrak{A}}\rangle^*\operatorname{up}^{(\mathfrak{A};\,\mathfrak{Z}_0)}a&\sqsubseteq\\ \bigsqcap\left\langle\bigsqcup\circ\langle\alpha\rangle^*\right\rangle^*\langle\operatorname{atoms}^{\mathfrak{A}}\rangle^*\{\{a\}\}&=\\ \bigsqcap\left\{\left(\bigsqcup\circ\langle\alpha\rangle^*\right)\{a\}\right\}&=\\ \bigsqcap\left\{\bigsqcup\langle\alpha\rangle^*\{a\}\right\}&=\\ \bigsqcap\left\{\bigsqcup\{\alpha a\}\right\}&=\\ \bigsqcap\{\alpha a\}&=\alpha a.\end{aligned}$$

Finally,

$$\alpha a=\bigsqcap\left\langle\bigsqcup\circ\langle\alpha\rangle^*\circ\operatorname{atoms}^{\mathfrak{A}}\right\rangle^*\operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}a=\bigsqcap\langle\alpha'\rangle^*\operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}a=\langle f\rangle a,$$

so $\langle f\rangle$ is a continuation of α.

2°. Consider the relation $\delta'\in\mathscr{P}(\mathfrak{Z}_0\times\mathfrak{Z}_1)$ defined by the formula (for every $X\in\mathfrak{Z}_0$, $Y\in\mathfrak{Z}_1$)

$$X\;\delta'\;Y\Leftrightarrow\exists x\in\operatorname{atoms}^{\mathfrak{A}}X,y\in\operatorname{atoms}^{\mathfrak{B}}Y:x\;\delta\;y.$$

Obviously $\neg(X\;\delta'\;\bot^{\mathfrak{Z}_1})$ and $\neg(\bot^{\mathfrak{Z}_0}\;\delta'\;Y)$.

$$\begin{aligned}I\sqcup J\;\delta'\;Y&\Leftrightarrow\\ \exists x\in\operatorname{atoms}^{\mathfrak{A}}(I\sqcup J),y\in\operatorname{atoms}^{\mathfrak{B}}Y:x\;\delta\;y&\Leftrightarrow\\ \exists x\in\operatorname{atoms}^{\mathfrak{A}}I\cup\operatorname{atoms}^{\mathfrak{A}}J,y\in\operatorname{atoms}^{\mathfrak{B}}Y:x\;\delta\;y&\Leftrightarrow\\ \exists x\in\operatorname{atoms}^{\mathfrak{A}}I,y\in\operatorname{atoms}^{\mathfrak{B}}Y:x\;\delta\;y\vee\exists x\in\operatorname{atoms}^{\mathfrak{A}}J,y\in\operatorname{atoms}^{\mathfrak{B}}Y:x\;\delta\;y&\Leftrightarrow\\ I\;\delta'\;Y\vee J\;\delta'\;Y;\end{aligned}$$

similarly $X\;\delta'\;I\sqcup J\Leftrightarrow X\;\delta'\;I\vee X\;\delta'\;J$. Let's continue δ' till a funcoid f (by the theorem 1082):

$$\mathcal{X}\;[f]\;\mathcal{Y}\Leftrightarrow\forall X\in\operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X},Y\in\operatorname{up}^{(\mathfrak{B};\mathfrak{Z}_1)}\mathcal{Y}:X\;\delta'\;Y.$$

The reverse of (20) implication is trivial, so

$$\forall X\in\operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}a,Y\in\operatorname{up}^{(\mathfrak{B};\mathfrak{Z}_1)}b\exists x\in\operatorname{atoms}^{\mathfrak{A}}X,y\in\operatorname{atoms}^{\mathfrak{B}}Y:x\;\delta\;y\Leftrightarrow a\;\delta\;b.$$

$$\begin{aligned}\forall X\in\operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}a,Y\in\operatorname{up}^{(\mathfrak{B};\mathfrak{Z}_1)}b\exists x\in\operatorname{atoms}^{\mathfrak{A}}X,y\in\operatorname{atoms}^{\mathfrak{B}}Y:x\;\delta\;y&\Leftrightarrow\\ \forall X\in\operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}a,Y\in\operatorname{up}^{(\mathfrak{B};\mathfrak{Z}_1)}b:X\;\delta'\;Y&\Leftrightarrow\\ a\;[f]\;b.\end{aligned}$$

So $a\;\delta\;b\Leftrightarrow a\;[f]\;b$, that is $[f]$ is a continuation of δ.

□

THEOREM 1115. Let $(\mathfrak{A};\mathfrak{Z}_0)$ and $(\mathfrak{B};\mathfrak{Z}_1)$ be primary filtrators over boolean lattices. If $R\in\mathscr{P}\mathsf{FCD}(\mathfrak{A};\mathfrak{B})$ and $x\in\operatorname{atoms}^{\mathfrak{A}}$, $y\in\operatorname{atoms}^{\mathfrak{B}}$, then

1°. $\langle\bigsqcap R\rangle x=\bigsqcap\left\{\frac{\langle f\rangle x}{f\in R}\right\}$;

2°. $x\;[\bigsqcap R]\;y\Leftrightarrow\forall f\in R:x\;[f]\;y$.

PROOF.