DEFINITION 1103. *Domain* of a pointfree funcoid f is defined by the formula $\operatorname{dom} f = \operatorname{im} f^{-1}$.

PROPOSITION 1104. $\langle f\rangle \operatorname{dom} f = \operatorname{im} f$ if f is a pointfree funcoid and $\operatorname{Src} f$ has greatest element $\top$ and $\operatorname{Dst} f$ is a separable poset.

PROOF. For every $y \in \operatorname{Dst} f$

$$y \not\asymp \langle f\rangle \operatorname{dom} f \Leftrightarrow \operatorname{dom} f \not\asymp \langle f^{-1}\rangle y \Leftrightarrow \langle f^{-1}\rangle \top \not\asymp \langle f^{-1}\rangle y \Leftrightarrow \langle f^{-1}\rangle y \neq \bot \Leftrightarrow 1 \not\asymp \langle f^{-1}\rangle y \Leftrightarrow y \not\asymp \langle f\rangle \top \Leftrightarrow y \not\asymp \operatorname{im} f.$$

So $\langle f\rangle \operatorname{dom} f = \operatorname{im} f$ by separability of $\operatorname{Dst} f$. □

PROPOSITION 1105. $\langle f\rangle x = \langle f\rangle (x \sqcap \operatorname{dom} f)$ whenever $\operatorname{dom} f$ is defined, for every $x \in \operatorname{Src} f$ for a pointfree funcoid f whose source is a meet-semilattice with least element and destination is a separable poset with least element.

PROOF. For every $y \in \operatorname{Dst} f$ we have

$$y \not\asymp \langle f\rangle (x \sqcap \operatorname{dom} f) \Leftrightarrow x \sqcap \operatorname{dom} f \sqcap \langle f^{-1}\rangle y \neq \bot^{\operatorname{Src} f} \Leftrightarrow x \sqcap \operatorname{im} f^{-1} \sqcap \langle f^{-1}\rangle y \neq \bot^{\operatorname{Src} f} \Leftrightarrow x \sqcap \langle f^{-1}\rangle y \neq \bot^{\operatorname{Src} f} \Leftrightarrow y \not\asymp \langle f\rangle x.$$

Thus $\langle f\rangle x = \langle f\rangle (x \sqcap \operatorname{dom} f)$ by separability of $\operatorname{Dst} f$. □

PROPOSITION 1106. $x \not\asymp \operatorname{dom} f \Leftrightarrow (\langle f\rangle x$ is not least) for every pointfree funcoid f and $x \in \operatorname{Src} f$ if $\operatorname{Dst} f$ has greatest element $\top$ and $\operatorname{Src} f$ is a separable poset.

PROOF. $x \not\asymp \operatorname{dom} f \Leftrightarrow x \not\asymp \langle f^{-1}\rangle \top^{\operatorname{Dst} f} \Leftrightarrow \top^{\operatorname{Dst} f} \not\asymp \langle f\rangle x \Leftrightarrow$ ($\langle f\rangle x$ is not least). □

COROLLARY 1107. $\operatorname{dom} f = \bigsqcup \left\{ \frac{a \in \operatorname{atoms}^{\operatorname{Src} f}}{\langle f\rangle a \neq \bot^{\operatorname{Dst} f}} \right\}$ for every pointfree funcoid f whose destination is a bounded poset and source is a separable atomistic meet-semilattice.

PROOF. For every $a \in \operatorname{atoms}^{\operatorname{Src} f}$ we have

$$a \not\asymp \operatorname{dom} f \Leftrightarrow a \not\asymp \langle f^{-1}\rangle \top^{\operatorname{Dst} f} \Leftrightarrow \top^{\operatorname{Dst} f} \not\asymp \langle f\rangle a \Leftrightarrow \langle f\rangle a \neq \bot^{\operatorname{Dst} f}.$$

So $\operatorname{dom} f = \bigsqcup \left\{ \frac{a \in \operatorname{atoms}^{\operatorname{Src} f}}{a \not\asymp \operatorname{dom} f} \right\} = \bigsqcup \left\{ \frac{a \in \operatorname{atoms}^{\operatorname{Src} f}}{\langle f\rangle a \neq \bot^{\operatorname{Dst} f}} \right\}$. □

PROPOSITION 1108. $\operatorname{dom}(f|_a) = a \sqcap \operatorname{dom} f$ for every pointfree funcoid f and $a \in \operatorname{Src} f$ where $\operatorname{Src} f$ is a separable meet-semilattice and $\operatorname{Dst} f$ has greatest element.

PROOF.

$$\operatorname{dom}(f|_a) = \operatorname{im}(\operatorname{id}_a^{\mathsf{FCD}(\operatorname{Src} f)} \circ f^{-1}) = \left\langle \operatorname{id}_a^{\mathsf{FCD}(\operatorname{Src} f)} \right\rangle \langle f^{-1}\rangle \top^{\operatorname{Dst} f} = a \sqcap \langle f^{-1}\rangle \top^{\operatorname{Dst} f} = a \sqcap \operatorname{dom} f.$$

□

PROPOSITION 1109. For every composable pointfree funcoids f and g where the posets $\operatorname{Src} f$ and $\operatorname{Dst} f = \operatorname{Src} g$ have greatest elements and $\operatorname{Dst} f$ and $\operatorname{Dst} g$ are separable:

1°. If $\operatorname{im} f \sqsupseteq \operatorname{dom} g$ then $\operatorname{im}(g \circ f) = \operatorname{im} g$.
2°. If $\operatorname{im} f \sqsubseteq \operatorname{dom} g$ then $\operatorname{dom}(g \circ f) = \operatorname{dom} g$.

PROOF.

1°. $\operatorname{im}(g \circ f) = \langle g \circ f\rangle \top^{\operatorname{Src} f} = \langle g\rangle \langle f\rangle \top^{\operatorname{Src} f} = \langle g\rangle \operatorname{im} f = \langle g\rangle \operatorname{dom} g = \langle g\rangle \top^{\operatorname{Src} g} = \operatorname{im} g$.