

2°.  $\alpha X \stackrel{\text{def}}{=} \sqcup \left\{ \frac{\langle f \rangle X}{f \in R} \right\}$  (by corollary 374 all joins on  $\mathfrak{B}$  exist). We have  $\alpha \perp^{\mathfrak{A}} = \perp^{\mathfrak{B}}$ ;

$$\begin{aligned} \alpha(I \sqcup^{\mathfrak{A}_0} J) &= \\ \sqcup \left\{ \frac{\langle f \rangle (I \sqcup^{\mathfrak{A}_0} J)}{f \in R} \right\} &= \\ \sqcup \left\{ \frac{\langle f \rangle (I \sqcup^{\mathfrak{A}} J)}{f \in R} \right\} &= \\ \sqcup \left\{ \frac{\langle f \rangle I \sqcup^{\mathfrak{B}} \langle f \rangle J}{f \in R} \right\} &= \\ \sqcup \left\{ \frac{\langle f \rangle I}{f \in R} \right\} \sqcup^{\mathfrak{B}} \sqcup \left\{ \frac{\langle f \rangle J}{f \in R} \right\} &= \\ \alpha I \sqcup^{\mathfrak{B}} \alpha J & \end{aligned}$$

(used theorem 1071). By theorem 1082 the function  $\alpha$  can be continued to  $\langle h \rangle$  for an  $h \in \text{FCD}(\mathfrak{A}; \mathfrak{B})$ . Obviously

$$\forall f \in R : h \sqsupseteq f. \quad (17)$$

And  $h$  is the least element of  $\text{FCD}(\mathfrak{A}; \mathfrak{B})$  for which the condition (17) holds. So  $h = \sqcup R$ .

1°.

$$\begin{aligned} X \left[ \sqcup R \right] Y &\Leftrightarrow \\ Y \sqcap^{\mathfrak{B}} \left\langle \sqcup R \right\rangle X &\neq \perp^{\mathfrak{B}} \Leftrightarrow \\ Y \sqcap^{\mathfrak{B}} \sqcup \left\{ \frac{\langle f \rangle X}{f \in R} \right\} &\neq \perp^{\mathfrak{B}} \Leftrightarrow \\ \exists f \in R : Y \sqcap^{\mathfrak{B}} \langle f \rangle X &\neq \perp^{\mathfrak{B}} \Leftrightarrow \\ \exists f \in R : X [f] Y & \end{aligned}$$

(used theorem 385). □

COROLLARY 1090. If  $(\mathfrak{A}; \mathfrak{A}_0)$  and  $(\mathfrak{B}; \mathfrak{B}_1)$  are primary filtrators over boolean lattices then  $\text{FCD}(\mathfrak{A}; \mathfrak{B})$  is a complete lattice.

PROOF. Apply [26]. □

THEOREM 1091. Let  $\mathfrak{A}$  and  $\mathfrak{B}$  be starrish join-semilattices. Then for  $f, g \in \text{FCD}(\mathfrak{A}; \mathfrak{B})$ :

- 1°.  $\langle f \sqcup g \rangle x = \langle f \rangle x \sqcup \langle g \rangle x$  for every  $x \in \mathfrak{A}$ ;
- 2°.  $[f \sqcup g] = [f] \cup [g]$ .

PROOF. □

1°. Let  $\alpha \mathcal{X} \stackrel{\text{def}}{=} \langle f \rangle x \sqcup \langle g \rangle x$ ;  $\beta \mathcal{Y} \stackrel{\text{def}}{=} \langle f^{-1} \rangle y \sqcup \langle g^{-1} \rangle y$  for every  $x \in \mathfrak{A}$ ,  $y \in \mathfrak{B}$ . Then

$$\begin{aligned} y \not\prec^{\mathfrak{B}} \alpha x &\Leftrightarrow \\ y \not\prec \langle f \rangle x \vee y \not\prec \langle g \rangle x &\Leftrightarrow \\ x \not\prec \langle f^{-1} \rangle y \vee x \not\prec \langle g^{-1} \rangle y &\Leftrightarrow \\ x \not\prec \langle f^{-1} \rangle y \sqcup \langle g^{-1} \rangle y &\Leftrightarrow \\ x \not\prec \beta y. & \end{aligned}$$