Taking into account properties of generalized filter bases:

$$\langle f\rangle \bigsqcap^{\operatorname{Src} f} S =$$
$$\bigsqcap^{\operatorname{Dst} f} \langle\langle f\rangle\rangle^* \operatorname{up} \bigsqcap S =$$
$$\bigsqcap^{\operatorname{Dst} f} \langle\langle f\rangle\rangle^* \left\{ \frac{X}{\exists \mathcal{P} \in S : X \in \operatorname{up} \mathcal{P}} \right\} =$$
$$\bigsqcap^{\operatorname{Dst} f} \left\{ \frac{\langle f\rangle^* X}{\exists \mathcal{P} \in S : X \in \operatorname{up} \mathcal{P}} \right\} \sqsupseteq \text{ (because } \operatorname{Dst} f \text{ is a separable poset)}$$
$$\bigsqcap^{\operatorname{Dst} f} \left\{ \frac{\langle f\rangle \mathcal{P}}{\mathcal{P} \in S} \right\} =$$
$$\bigsqcap^{\operatorname{Dst} f} \langle\langle f\rangle\rangle^* S.$$

□

15.4. The order of pointfree funcoids

Definition 1084. The order of pointfree funcoids $\mathsf{FCD}(\mathfrak{A}; \mathfrak{B})$ is defined by the formula:

$$f \sqsubseteq g \Leftrightarrow \forall x \in \mathfrak{A} : \langle f\rangle x \sqsubseteq \langle g\rangle x \wedge \forall y \in \mathfrak{B} : \left\langle f^{-1}\right\rangle y \sqsubseteq \left\langle g^{-1}\right\rangle y.$$

Proposition 1085. It is really a partial order on the set $\mathsf{FCD}(\mathfrak{A}; \mathfrak{B})$.

Proof.

Reflexivity. Obvious.
Transitivity. It follows from transitivity of the order relations on $\mathfrak{A}$ and $\mathfrak{B}$.
Antisymmetry. It follows from antisymmetry of the order relations on $\mathfrak{A}$ and $\mathfrak{B}$.

□

Remark 1086. It is enough to define order of pointfree funcoids on every set $\mathsf{FCD}(\mathfrak{A}; \mathfrak{B})$ where $\mathfrak{A}$ and $\mathfrak{B}$ are posets. We do not need to compare pointfree funcoids with different sources or destinations.

Obvious 1087. $f \sqsubseteq g \Rightarrow [f] \subseteq [g]$ for every $f, g \in \mathsf{FCD}(\mathfrak{A}; \mathfrak{B})$ for every posets $\mathfrak{A}$ and $\mathfrak{B}$.

Theorem 1088. If $\mathfrak{A}$ and $\mathfrak{B}$ are separable posets then $f \sqsubseteq g \Leftrightarrow [f] \subseteq [g]$.

Proof. From the theorem 1068. □

Theorem 1089. Let $(\mathfrak{A}; \mathfrak{Z}_0)$ and $(\mathfrak{B}; \mathfrak{Z}_1)$ be primary filtrators over boolean lattices. Then for $R \in \mathscr{P}\mathsf{FCD}(\mathfrak{A}; \mathfrak{B})$ and $X \in \mathfrak{Z}_0$, $Y \in \mathfrak{Z}_1$ we have:

1°. $X \left[\bigsqcup R\right] Y \Leftrightarrow \exists f \in R : X \left[f\right] Y$;
2°. $\langle \bigsqcup R\rangle X = \left\{ \frac{\langle f\rangle X}{f \in R} \right\}$.

Proof.