Combining the equivalencies we get $\mathcal{Y} \sqcap \alpha'\mathcal{X} \neq \bot^{\mathfrak{B}} \Leftrightarrow \mathcal{X}\ \delta'\ \mathcal{Y}$. Analogously $\mathcal{X} \sqcap \beta'\mathcal{Y} \neq \bot^{\mathfrak{A}} \Leftrightarrow \mathcal{X}\ \delta'\ \mathcal{Y}$. So $\mathcal{Y} \sqcap \alpha'\mathcal{X} \neq \bot^{\mathfrak{B}} \Leftrightarrow \mathcal{X} \sqcap \beta'\mathcal{Y} \neq \bot^{\mathfrak{A}}$, that is $(\mathfrak{A}; \mathfrak{B}; \alpha'; \beta')$ is a pointfree funcoid. From the formula $\mathcal{Y} \sqcap \alpha'\mathcal{X} \neq \bot^{\mathfrak{B}} \Leftrightarrow \mathcal{X}\ \delta'\ \mathcal{Y}$ it follows that $[(\mathfrak{A}; \mathfrak{B}; \alpha'; \beta')]$ is a continuation of δ.

1°. Let define the relation $\delta \in \mathscr{P}(\mathfrak{Z}_0 \times \mathfrak{Z}_1)$ by the formula $X\ \delta\ Y \Leftrightarrow Y \sqcap^{\mathfrak{B}} \alpha X \neq \bot^{\mathfrak{B}}$.

That $\neg(\bot^{\mathfrak{Z}_0}\ \delta\ I')$ and $\neg(I\ \delta\ \bot^{\mathfrak{Z}_1})$ is obvious. We have

$$\begin{aligned}
&K\ \delta\ I' \sqcup^{\mathfrak{Z}_1} J' \Leftrightarrow\\
&(I' \sqcup^{\mathfrak{Z}_1} J') \sqcap^{\mathfrak{B}} \alpha K \neq \bot^{\mathfrak{B}} \Leftrightarrow\\
&(I' \sqcup^{\mathfrak{B}} J') \sqcap \alpha K \neq \bot^{\mathfrak{B}} \Leftrightarrow\\
&(I' \sqcap^{\mathfrak{B}} \alpha K) \sqcup (J' \sqcap^{\mathfrak{B}} \alpha K) \neq \bot^{\mathfrak{B}} \Leftrightarrow\\
&I' \sqcap^{\mathfrak{B}} \alpha K \neq 0^{\mathfrak{B}} \vee J' \sqcap^{\mathfrak{B}} \alpha K \neq \bot^{\mathfrak{B}} \Leftrightarrow\\
&K\ \delta\ I' \vee K\ \delta\ J'
\end{aligned}$$

and

$$\begin{aligned}
&I \sqcup^{\mathfrak{Z}_0} J\ \delta\ K' \Leftrightarrow\\
&K' \sqcap^{\mathfrak{B}} \alpha(I \sqcup^{\mathfrak{Z}_0} J) \neq \bot^{\mathfrak{B}} \Leftrightarrow\\
&K' \sqcap^{\mathfrak{B}} (\alpha I \sqcup \alpha J) \neq \bot^{\mathfrak{B}} \Leftrightarrow\\
&(K' \sqcap^{\mathfrak{B}} \alpha I) \sqcup (K' \sqcap^{\mathfrak{B}} \alpha J) \neq \bot^{\mathfrak{B}} \Leftrightarrow\\
&K' \sqcap^{\mathfrak{B}} \alpha I \neq 0^{\mathfrak{B}} \vee K' \sqcap^{\mathfrak{B}} \alpha J \neq \bot^{\mathfrak{B}} \Leftrightarrow\\
&I\ \delta\ K' \vee J\ \delta\ K'.
\end{aligned}$$

That is the formulas (15) are true.

Accordingly the above δ can be continued to the relation $[f]$ for some $f \in \mathsf{FCD}(\mathfrak{A}; \mathfrak{B})$.

$\forall X \in \mathfrak{Z}_0, Y \in \mathfrak{Z}_1 : (Y \sqcap^{\mathfrak{B}} \langle f\rangle X \neq \bot^{\mathfrak{B}} \Leftrightarrow X\ [f]\ Y \Leftrightarrow Y \sqcap^{\mathfrak{B}} \alpha X \neq \bot^{\mathfrak{B}})$, consequently $\forall X \in \mathfrak{Z}_0 : \alpha X = \langle f\rangle X$ because our filtrator is with separable core. So $\langle f\rangle$ is a continuation of α.

□

Proposition 1083. Let $(\operatorname{Src} f; \mathfrak{Z}_0)$ be a primary filtrator over a bounded distributive lattice and $(\operatorname{Dst} f; \mathfrak{Z}_1)$ is a primary filtrator over a boolean lattice. If S is a generalized filter base on $\operatorname{Src} f$ then $\langle f\rangle \bigsqcap^{\operatorname{Src} f} S = \bigsqcap^{\operatorname{Dst} f} \langle\langle f\rangle\rangle^* S$ for every pointfree funcoid f.

Proof. First the meets $\bigsqcap^{\operatorname{Src} f} S$ and $\bigsqcap^{\operatorname{Dst} f} \langle\langle f\rangle\rangle^* S$ exist by corollary 374.

$(\operatorname{Src} f; \mathfrak{Z}_0)$ is a finitely meet-closed filtrator by proposition 364 and with separable core by theorem 379; thus we can apply theorem 1081 (up $x \neq \emptyset$ is obvious).

$\langle f\rangle \bigsqcap^{\operatorname{Src} f} S \sqsubseteq \langle f\rangle X$ for every $X \in S$ because $\operatorname{Dst} f$ is separable by obvious 403 and thus $\langle f\rangle \bigsqcap^{\operatorname{Src} f} S \sqsubseteq \bigsqcap^{\operatorname{Dst} f} \langle\langle f\rangle\rangle^* S$.