So $\langle f\rangle x = \bigsqcap^{\operatorname{Dst} f}\langle\langle f\rangle\rangle^* \operatorname{up}^{(\operatorname{Src} f;\mathfrak{Z}_0)} x$ because $\operatorname{Dst} f$ is separable (obvious 403 and the fact that $\mathfrak{Z}_1$ is a boolean lattice). □

THEOREM 1082. Let $(\mathfrak{A};\mathfrak{Z}_0)$ and $(\mathfrak{B};\mathfrak{Z}_1)$ be primary filtrators over boolean lattices.

1°. A function $\alpha \in \mathfrak{B}^{\mathfrak{Z}_0}$ conforming to the formulas (for every $I, J \in \mathfrak{Z}_0$)

$$\alpha\bot^{\mathfrak{Z}_0} = \bot^{\mathfrak{B}}, \quad \alpha(I \sqcup J) = \alpha I \sqcup \alpha J$$

can be continued to the function $\langle f\rangle$ for a unique $f \in \mathsf{FCD}(\mathfrak{A};\mathfrak{B})$;

$$\langle f\rangle\mathcal{X} = \bigsqcap^{\mathfrak{B}}\langle\alpha\rangle^* \operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X} \tag{14}$$

for every $\mathcal{X} \in \mathfrak{A}$.

2°. A relation $\delta \in \mathscr{P}(\mathfrak{Z}_0 \times \mathfrak{Z}_1)$ conforming to the formulas (for every $I, J, K \in \mathfrak{Z}_0$ and $I', J', K' \in \mathfrak{Z}_1$)

$$\begin{aligned} &\neg(\bot^{\mathfrak{Z}_0}\ \delta\ I'), \quad I \sqcup J\ \delta\ K' \Leftrightarrow I\ \delta\ K' \vee J\ \delta\ K', \\ &\neg(I\ \delta\ \bot^{\mathfrak{Z}_1}), \quad K\ \delta\ I' \sqcup J' \Leftrightarrow K\ \delta\ I' \vee K\ \delta\ J' \end{aligned} \tag{15}$$

can be continued to the relation $[f]$ for a unique $f \in \mathsf{FCD}(\mathfrak{A};\mathfrak{B})$;

$$\mathcal{X}\ [f]\ \mathcal{Y} \Leftrightarrow \forall X \in \operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X}, Y \in \operatorname{up}^{(\mathfrak{B};\mathfrak{Z}_1)}\mathcal{Y} : X\ \delta\ Y \tag{16}$$

for every $\mathcal{X} \in \mathfrak{A}$, $\mathcal{Y} \in \mathfrak{B}$.

PROOF. Existence of no more than one such pointfree funcoids and formulas (14) and (16) follow from two previous theorems.

2°. $\left\{\frac{Y\in\mathfrak{Z}_1}{X\delta Y}\right\}$ is obviously a free star for every $X \in \mathfrak{Z}_0$. By properties of filters on boolean lattices, there exist a unique filter $\alpha X$ such that $\partial(\alpha X) = \left\{\frac{Y\in\mathfrak{Z}_1}{X\delta Y}\right\}$ for every $X \in \mathfrak{Z}_0$. Thus $\alpha \in \mathfrak{B}^{\mathfrak{Z}_0}$. Similarly it can be defined $\beta \in \mathfrak{A}^{\mathfrak{Z}_1}$ by the formula $\partial(\beta Y) = \left\{\frac{Y\in\mathfrak{Z}_1}{X\delta Y}\right\}$. Let's continue the functions $\alpha$ and $\beta$ to $\alpha' \in \mathfrak{B}^{\mathfrak{A}}$ and $\beta' \in \mathfrak{A}^{\mathfrak{B}}$ by the formulas

$$\alpha'\mathcal{X} = \bigsqcap^{\mathfrak{B}}\langle\alpha\rangle^* \operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X} \quad \text{and} \quad \beta'\mathcal{Y} = \bigsqcap^{\mathfrak{A}}\langle\beta\rangle^* \operatorname{up}^{(\mathfrak{B};\mathfrak{Z}_1)}\mathcal{Y}$$

and $\delta$ to $\delta' \in \mathscr{P}(\mathfrak{A} \times \mathfrak{B})$ by the formula

$$\mathcal{X}\ \delta'\ \mathcal{Y} \Leftrightarrow \forall X \in \operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X}, Y \in \operatorname{up}^{(\mathfrak{B};\mathfrak{Z}_1)}\mathcal{Y} : \mathcal{X}\ \delta\ \mathcal{Y}.$$

$\mathcal{Y} \sqcap \alpha'\mathcal{X} \neq \bot^{\mathfrak{B}} \Leftrightarrow \mathcal{Y} \sqcap \bigsqcap\langle\alpha\rangle^* \operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X} \neq \bot^{\mathfrak{B}} \Leftrightarrow \bigsqcap\langle\mathcal{Y}\sqcap\rangle^*\langle\alpha\rangle^* \operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X} \neq \bot^{\mathfrak{B}}$. Let's prove that

$$W = \langle\mathcal{Y}\sqcap\rangle^*\langle\alpha\rangle^* \operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X}$$

is a generalized filter base: To prove it is enough to show that $\langle\alpha\rangle^* \operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X}$ is a generalized filter base.

If $\mathcal{A}, \mathcal{B} \in \langle\alpha\rangle^* \operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X}$ then exist $X_1, X_2 \in \operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X}$ such that $\mathcal{A} = \alpha X_1$ and $\mathcal{B} = \alpha X_2$. Then $\alpha(X_1 \sqcap^{\mathfrak{Z}_0} X_2) \in \langle\alpha\rangle^* \operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X}$. So $\langle\alpha\rangle^* \operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X}$ is a generalized filter base and thus $W$ is a generalized filter base.

By properties of generalized filter bases, $\bigsqcap\langle\mathcal{Y}\sqcap\rangle^*\langle\alpha\rangle^* \operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X} \neq \bot^{\mathfrak{B}}$ is equivalent to

$$\forall X \in \operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X} : \mathcal{Y} \sqcap \alpha X \neq \bot^{\mathfrak{B}},$$

what is equivalent to

$$\begin{aligned} &\forall X \in \operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X}, Y \in \operatorname{up}^{(\mathfrak{B};\mathfrak{Z}_1)}\mathcal{Y} : Y \sqcap^{\mathfrak{B}} \alpha X \neq \bot^{\mathfrak{B}} \Leftrightarrow \\ &\forall X \in \operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X}, Y \in \operatorname{up}^{(\mathfrak{B};\mathfrak{Z}_1)}\mathcal{Y} : Y \in \partial(\alpha X) \Leftrightarrow \\ &\forall X \in \operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X}, Y \in \operatorname{up}^{(\mathfrak{B};\mathfrak{Z}_1)}\mathcal{Y} : X\ \delta\ Y. \end{aligned}$$