PROOF. Note that $\left\langle \mathrm{id}^{\mathsf{FCD}}_{\Omega(\mathfrak{U})} \right\rangle \mathcal{X} = \mathcal{X} \sqcap \Omega(\mathfrak{U})$ for every filter $\mathcal{X}$ on $\mathfrak{U}$.

Let $f = \mathrm{id}^{\mathsf{FCD}(\mathfrak{U})}$, $g = \uparrow^{\mathsf{FCD}(\mathfrak{U};\mathfrak{U})} ((\mathfrak{U} \times \mathfrak{U}) \setminus \mathrm{id}_{\mathfrak{U}})$.

Let $x$ be a non-trivial ultrafilter on $\mathfrak{U}$. If $X \in x$ then $\operatorname{card} X \geq 2$ (In fact, $X$ is infinite but we don't need this.) and consequently $\langle g \rangle^* X = \top^{\mathfrak{F}(\mathfrak{U})}$. Thus $\langle g \rangle x = \top^{\mathfrak{F}(\mathfrak{U})}$. Consequently

$$\langle f \sqcap g \rangle x = \langle f \rangle x \sqcap \langle g \rangle x = x \sqcap \top^{\mathfrak{F}(\mathfrak{U})} = x.$$

Also $\left\langle \mathrm{id}^{\mathsf{FCD}}_{\Omega(\mathfrak{U})} \right\rangle x = x \sqcap \Omega(\mathfrak{U}) = x$.

Let now $x$ be a trivial ultrafilter. Then $\langle f \rangle x = x$ and $\langle g \rangle x = \top^{\mathfrak{F}(\mathfrak{U})} \setminus x$. So

$$\langle f \sqcap g \rangle x = \langle f \rangle x \sqcap \langle g \rangle x = x \sqcap (\top^{\mathfrak{F}(\mathfrak{U})} \setminus x) = \bot^{\mathfrak{F}(\mathfrak{U})}.$$

Also $\left\langle \mathrm{id}^{\mathsf{FCD}}_{\Omega(\mathfrak{U})} \right\rangle x = x \sqcap \Omega(\mathfrak{U}) = \bot^{\mathfrak{F}(\mathfrak{U})}$.

So $\langle f \sqcap g \rangle x = \left\langle \mathrm{id}^{\mathsf{FCD}}_{\Omega(\mathfrak{U})} \right\rangle x$ for every ultrafilter $x$ on $\mathfrak{U}$. Thus $f \sqcap g = \mathrm{id}^{\mathsf{FCD}}_{\Omega(\mathfrak{U})}$. □

EXAMPLE 1036. There exist binary relations $f$ and $g$ such that $\uparrow^{\mathsf{FCD}(A;B)} f \sqcap \uparrow^{\mathsf{FCD}(A;B)} g \neq \uparrow^{\mathsf{FCD}(A;B)} (f \cap g)$ for some sets $A$, $B$ such that $f, g \subseteq A \times B$.

PROOF. From the proposition above. □

EXAMPLE 1037. There exists a principal funcoid which is not a complemented element of the lattice of funcoids.

PROOF. I will prove that quasi-complement of the funcoid $\mathrm{id}^{\mathsf{FCD}(\mathbb{N})}$ is not its complement. We have:

$$\begin{aligned}
(\mathrm{id}^{\mathsf{FCD}(\mathbb{N})})^* &= \\
\bigsqcup \left\{ \frac{c \in \mathsf{FCD}(\mathbb{N};\mathbb{N})}{c \asymp \mathrm{id}^{\mathsf{FCD}(\mathbb{N})}} \right\} &\sqsupseteq \\
\bigsqcup \left\{ \frac{\uparrow^{\mathbb{N}} \{\alpha\} \times^{\mathsf{FCD}} \uparrow^{\mathbb{N}} \{\beta\}}{\alpha, \beta \in \mathbb{N}, \uparrow^{\mathbb{N}} \{\alpha\} \times^{\mathsf{FCD}} \uparrow^{\mathbb{N}} \{\beta\} \asymp \mathrm{id}^{\mathsf{FCD}(\mathbb{N})}} \right\} &= \\
\bigsqcup \left\{ \frac{\uparrow^{\mathbb{N}} \{\alpha\} \times^{\mathsf{FCD}} \uparrow^{\mathbb{N}} \{\beta\}}{\alpha, \beta \in \mathbb{N}, \alpha \neq \beta} \right\} &= \\
\uparrow^{\mathsf{FCD}(\mathbb{N};\mathbb{N})} \bigsqcup \left\{ \frac{\{\alpha\} \times \{\beta\}}{\alpha, \beta \in \mathbb{N}, \alpha \neq \beta} \right\} &= \\
\uparrow^{\mathsf{FCD}(\mathbb{N};\mathbb{N})} (\mathbb{N} \times \mathbb{N} \setminus \mathrm{id}_{\mathbb{N}})
\end{aligned}$$

(used corollary 674). But by proved above $(\mathrm{id}^{\mathsf{FCD}(\mathbb{N})})^* \sqcap \mathrm{id}^{\mathsf{FCD}(\mathbb{N})} \neq \bot^{\mathfrak{F}(\mathbb{N})}$.FiXme: Stronger conjecture: $(\uparrow^{\mathsf{FCD}} f)^* = \bar{f}$ for every binary relation $f$. □

EXAMPLE 1038. There exists a funcoid $h$ such that $\mathrm{GR}\, h$ is not a filter.

PROOF. Consider the funcoid $h = \mathrm{id}^{\mathsf{FCD}}_{\Omega(\mathbb{N})}$. We have (from the proof of proposition 1035) that $f \in \mathrm{GR}\, h$ and $g \in \mathrm{GR}\, h$, but $f \cap g \notin \mathrm{GR}\, h$. □

EXAMPLE 1039. There exists a funcoid $h \neq \bot^{\mathsf{FCD}(A;B)}$ such that $(\mathsf{RLD})_{\mathrm{out}} h = \bot^{\mathsf{RLD}(A;B)}$.

PROOF. Consider $h = \mathrm{id}^{\mathsf{FCD}}_{\Omega(\mathbb{N})}$. By proved above $h = f \sqcap g$ where $f = \mathrm{id}^{\mathsf{FCD}(\mathbb{N})} = \uparrow^{\mathsf{FCD}(\mathbb{N};\mathbb{N})} \mathrm{id}_{\mathbb{N}}$, $g = \uparrow^{\mathsf{FCD}(\mathbb{N};\mathbb{N})} (\mathbb{N} \times \mathbb{N} \setminus \mathrm{id}_{\mathbb{N}})$.

We have $\mathrm{id}_{\mathbb{N}}, \mathbb{N} \times \mathbb{N} \setminus \mathrm{id}_{\mathbb{N}} \in \mathrm{GR}\, h$.

So

$$(\mathsf{RLD})_{\mathrm{out}} h = \bigsqcap \left\langle \uparrow^{\mathsf{RLD}(\mathbb{N};\mathbb{N})} \right\rangle^* \mathrm{GR}\, h \sqsubseteq \uparrow^{\mathsf{RLD}(\mathbb{N};\mathbb{N})} (\mathrm{id}_{\mathbb{N}} \cap (\mathbb{N} \times \mathbb{N} \setminus \mathrm{id}_{\mathbb{N}})) = \bot^{\mathsf{RLD}(\mathbb{N};\mathbb{N})};$$