

PROOF. Take  $\mathbb{R}_+ = \left\{ \frac{x \in \mathbb{R}}{x > 0} \right\}$ ,  $\mathcal{A} = \Delta$ ,  $T = \left\{ \frac{\uparrow \{x\}}{x \in \mathbb{R}_+} \right\}$  where  $\uparrow = \uparrow^{\mathbb{R}}$ .

$$\bigsqcup T = \uparrow \mathbb{R}_+; \mathcal{A} \times^{\text{RLD}} \bigsqcup T = \Delta \times^{\text{RLD}} \uparrow \mathbb{R}_+.$$

$$\bigsqcup \left\{ \frac{A \times^{\text{RLD}} B}{B \in T} \right\} = \bigsqcup \left\{ \frac{\Delta \times^{\text{RLD}} \uparrow \{x\}}{x \in \mathbb{R}_+} \right\}.$$

We'll prove that  $\bigsqcup \left\{ \frac{\Delta \times^{\text{RLD}} \uparrow \{x\}}{x \in \mathbb{R}_+} \right\} \neq \Delta \times^{\text{RLD}} \uparrow \mathbb{R}_+$ .

$$\text{Consider } K = \bigcup \left\{ \frac{\{x\} \times (-1/x; 1/x)}{x \in \mathbb{R}_+} \right\}.$$

$K \in \text{GR}(\Delta \times^{\text{RLD}} \uparrow \{x\})$  and thus  $K \in \text{GR} \bigsqcup \left\{ \frac{\Delta \times^{\text{RLD}} \uparrow \{x\}}{x \in \mathbb{R}_+} \right\}$ . But  $K \notin \text{GR}(\Delta \times^{\text{RLD}} \uparrow \mathbb{R}_+)$ .  $\square$

**THEOREM 1029.** For a filter  $a$  we have  $a \times^{\text{RLD}} a \sqsubseteq \text{id}^{\text{RLD}(\text{Base}(a))}$  only in the case if  $a = \perp^{\mathfrak{F}(\text{Base}(a))}$  or  $a$  is a trivial ultrafilter.

PROOF. If  $a \times^{\text{RLD}} a \sqsubseteq \text{id}^{\text{RLD}(\text{Base}(a))}$  then there exists  $m \in \text{GR}(a \times^{\text{RLD}} a)$  such that  $m \sqsubseteq \text{id}_{\text{Base}(a)}$ . Consequently there exist  $A, B \in \text{GR} a$  such that  $A \times B \sqsubseteq \text{id}_{\text{Base}(a)}$  what is possible only in the case when  $\uparrow^{\text{Base}(a)} A = \uparrow^{\text{Base}(a)} B = a$  is trivial a ultrafilter or the least filter.  $\square$

**COROLLARY 1030.** Reloidal product of a non-trivial atomic filter with itself is non-atomic.

PROOF. Obviously  $(a \times^{\text{RLD}} a) \sqcap \text{id}^{\text{RLD}(\text{Base}(a))} \neq \perp^{\mathfrak{F}(\text{Base}(a))}$  and  $(a \times^{\text{RLD}} a) \sqcap \text{id}^{\text{RLD}(\text{Base}(a))} \sqsubset a \times^{\text{RLD}} a$ .  $\square$

**EXAMPLE 1031.** There exist two atomic reloids whose composition is non-atomic and non-empty.

PROOF. Let  $a$  be a non-trivial ultrafilter on  $\mathbb{N}$  and  $x \in \mathbb{N}$ . Then

$$\begin{aligned} (a \times^{\text{RLD}} \uparrow^{\mathbb{N}} \{x\}) \circ (\uparrow^{\mathbb{N}} \{x\} \times^{\text{RLD}} a) &= \bigsqcap \left\{ \frac{\uparrow^{\text{RLD}(\mathbb{N}; \mathbb{N})} ((A \times \{x\}) \circ (\{x\} \times A))}{A \in a} \right\} = \\ &= \bigsqcap \left\{ \frac{\uparrow^{\text{RLD}(\mathbb{N}; \mathbb{N})} (A \times A)}{A \in a} \right\} = a \times^{\text{RLD}} a \end{aligned}$$

is non-atomic despite of  $a \times^{\text{RLD}} \uparrow^{\mathbb{N}} \{x\}$  and  $\uparrow^{\mathbb{N}} \{x\} \times^{\text{RLD}} a$  are atomic.  $\square$

**EXAMPLE 1032.** There exists non-monovalued atomic reloid.

PROOF. From the previous example it follows that the atomic reloid  $\uparrow^{\mathbb{N}} \{x\} \times^{\text{RLD}} a$  is not monovalued.  $\square$

**EXAMPLE 1033.** Non-convex reloids exist.

PROOF. Let  $a$  be a non-trivial ultrafilter. Then  $\text{id}_a^{\text{RLD}}$  is non-convex. This follows from the fact that only reloidal products which are below  $\text{id}^{\text{RLD}(\text{Base}(a))}$  are reloidal products of ultrafilters and  $\text{id}_a^{\text{RLD}}$  is not their join.  $\square$

**EXAMPLE 1034.**  $(\text{RLD})_{\text{in}} f \neq (\text{RLD})_{\text{out}} f$  for a functor  $f$ .

PROOF. Let  $f = \text{id}^{\text{FCD}(\mathbb{N})}$ . Then  $(\text{RLD})_{\text{in}} f = \bigsqcup \left\{ \frac{a \times^{\text{RLD}} a}{a \in \text{atoms}^{\mathfrak{F}(\mathbb{N})}} \right\}$  and  $(\text{RLD})_{\text{out}} f = \text{id}^{\text{RLD}(\mathbb{N})}$ . But we have shown above  $a \times^{\text{RLD}} a \not\sqsubseteq \text{id}^{\text{RLD}(\mathbb{N})}$  for non-trivial ultrafilter  $a$ , and so  $(\text{RLD})_{\text{in}} f \not\sqsubseteq (\text{RLD})_{\text{out}} f$ .  $\square$

**PROPOSITION 1035.**  $\text{id}^{\text{FCD}(\mathfrak{U})} \sqcap \uparrow^{\text{FCD}(\mathfrak{U}; \mathfrak{U})} ((\mathfrak{U} \times \mathfrak{U}) \setminus \text{id}_{\mathfrak{U}}) = \text{id}_{\Omega(\mathfrak{U})}^{\text{FCD}} \neq \perp^{\text{FCD}(\mathfrak{U}; \mathfrak{U})}$  for every infinite set  $\mathfrak{U}$ .