

LEMMA 1021.  $(\prod G) \circ f = \prod \left\{ \frac{g \circ f}{g \in G} \right\}$  if  $f$  is a monovalued principal reloid and  $G$  is a set of reloids (with matching sources and destinations).

PROOF. Let  $f = \uparrow^{\text{RLD}} \varphi$  for some monovalued **Rel**-morphism  $\varphi$ .  
 $(\prod G) \circ f = \prod \left\{ \frac{\uparrow^{\text{RLD}}(g \circ \varphi)}{g \in \text{xyGR} \prod G} \right\};$

$$\begin{aligned} & \text{GR} \prod \left\{ \frac{g \circ f}{g \in G} \right\} = \\ & \text{GR} \prod \left\{ \frac{\prod \left\{ \frac{\uparrow^{\text{RLD}}(\Gamma \circ \varphi)}{\Gamma \in \text{xyGR} g} \right\}}{g \in G} \right\} = \\ & \text{GR} \prod \bigcup \left\{ \frac{\left\{ \frac{\uparrow^{\text{RLD}}(\Gamma \circ \varphi)}{\Gamma \in \text{xyGR} g} \right\}}{g \in G} \right\} = \\ & \text{GR} \prod \left\{ \frac{\uparrow^{\text{RLD}}(\Gamma \circ \varphi)}{\Gamma \in \text{xyGR} \prod G} \right\} = \\ & \left\{ \frac{(\Gamma_0 \circ \varphi) \prod \cdots \prod (\Gamma_n \circ \varphi)}{\Gamma_i \in \bigcup G \text{ where } i = 0, \dots, n \text{ for } n \in \mathbb{N}} \right\} = \text{(proposition above)} \\ & \left\{ \frac{(\Gamma_0 \prod \cdots \prod \Gamma_n) \circ \varphi}{\Gamma_i \in \bigcup G \text{ where } i = 0, \dots, n \text{ for } n \in \mathbb{N}} \right\} = \\ & \left\{ \frac{\Gamma \circ \varphi}{\Gamma \in \text{xyGR} \prod G} \right\}. \end{aligned}$$

Thus  $(\prod G) \circ f = \prod \left\{ \frac{g \circ f}{g \in G} \right\}$ . □

THEOREM 1022.

- 1°. Monovalued reloids are metamonovalued.
- 2°. Injective reloids are metainjective.

PROOF. We will prove only the first, as the second is dual.

Let  $G$  be a set of reloids and  $f$  be a monovalued reloid.

Let  $f'$  be a principal monovalued continuation of  $f$  (so that  $f = f'|_{\text{dom } f}$ ).

By the lemma  $(\prod G) \circ f' = \prod \left\{ \frac{g \circ f'}{g \in G} \right\}$ . Restricting this equality to  $\text{dom } f$  we

get:  $(\prod G) \circ f = \prod \left\{ \frac{g \circ f}{g \in G} \right\}$ . □

CONJECTURE 1023. Every metamonovalued reloid is monovalued.